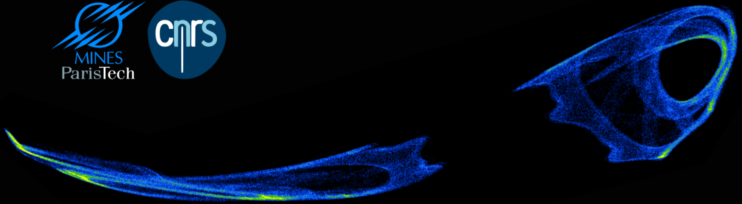
Wave propagation through contact-based elastically asymmetric materials

Vladislav A. Yastrebov

MINES ParisTech, PSL Research University, Centre des Matériaux, CNRS UMR 7633, Evry, France

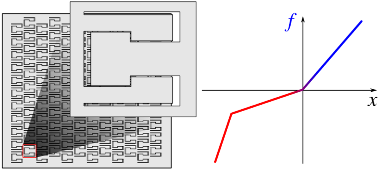


European Solid Mechanics Conference

July 2, 2018
Bologna, Italy

Outline

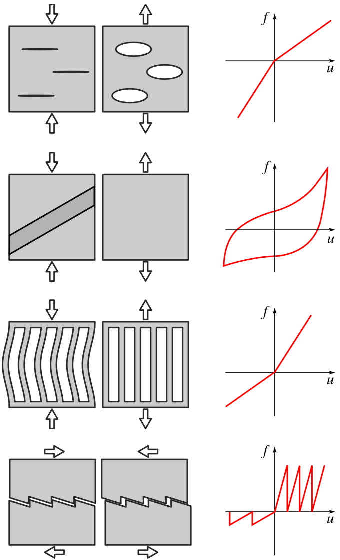
Introduction



Asymmetry in materials

Sources of asymmetry[A,B]**:**

- Micro-fractures
  concrete/rocks
- Local buckling/wrinkling
  fiber networks, ropes, membranes
- Phase transformations/twinning
  Mg alloys, Ti alloys
- Pressure dependent plasticity
  concrete/rocks/soils
- Sliding asymmetry
  patterned surfaces
- ★ Contacts
  granular matter, granular crystals



[A] Ambartsumyan (1965), Izv Academ Nauk USSR Mech

[B] Gibson, Ashby (1987), Cellular Solids, Cambridge UP

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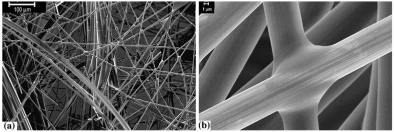

Fig. Carbon fiber network

[1] Mezeix, Bouvet, Huez, Poquillon (2009). J Mater Sci 44

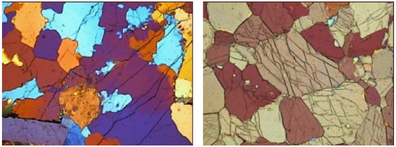

Fig. Microcracks in rocks (dolomite, granite)

[2] Obara (2007). Comput & Geosci 33

[3] Obara, Kozusnikova (2007). Computat Geosci 11

Fig. Torsional instability in multi-strand wires (ropes)

[4] www.industrialrope.com

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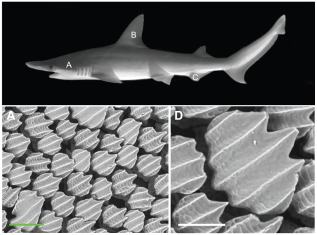

Fig. Shark skin
[5] Wen, Weaver, Lauder (2014). J Exp Biol 217

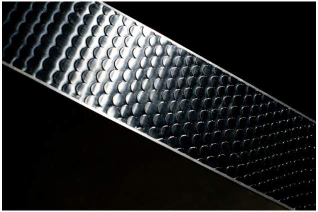

Fig. Fish-skin pattern on skitour skis
[6] Photo courtesy “Voile”

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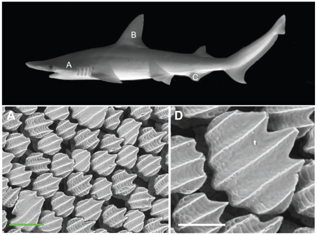
+: kirigamis from Katia Bertoldi's ope ning lecture.



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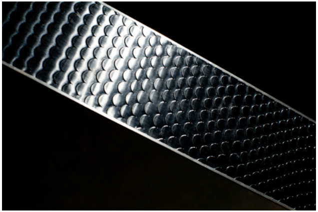

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Bibliographical sketch

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- Robert M. Jones (1971,1975, 1977)
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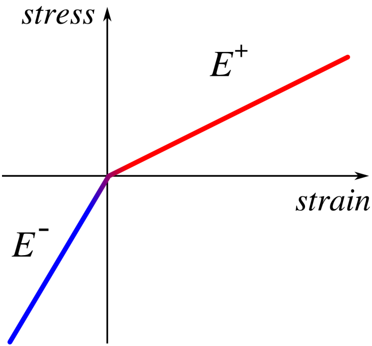

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Elastic properties depend on principal stresses

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{bmatrix} = \begin{bmatrix} S_{11}(\sigma_1) & S_{12}(\sigma_2) & S_{13}(\sigma_3)\\ S_{21}(\sigma_1) & S_{22}(\sigma_2) & S_{23}(\sigma_3)\\ S_{31}(\sigma_1) & S_{32}(\sigma_2) & S_{33}(\sigma_3)\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{bmatrix}$$

$$S_{11}(+) = S_{22}(+) = S_{33}(+) = \frac{1}{E^+}, \qquad S_{11}(-) = S_{22}(-) = S_{33}(-) = \frac{1}{E^-}$$

$$S_{12}(+) = S_{13}(+) = S_{23}(+) = -\frac{\nu^+}{E^+}, \qquad S_{12}(-) = S_{13}(-) = S_{23}(-) = -\frac{\nu^-}{E^-}$$

For symmetry

$$\nu^- E^+ = \nu^+ E^-$$

- Adapted variational framework[1].
- Anisotropic damage[2].

[1] Du, Guo (2014). J Mech Phys Solids 73
[2] Desmorat, Gatuingt, Ragueneau (2007). Eng Fract Mech 74

Contact-based architectured material

- Contact as a functional element

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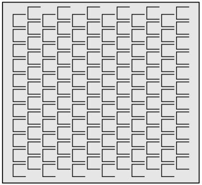
- Stiff in compression / soft in tension



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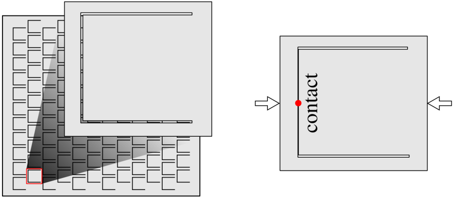

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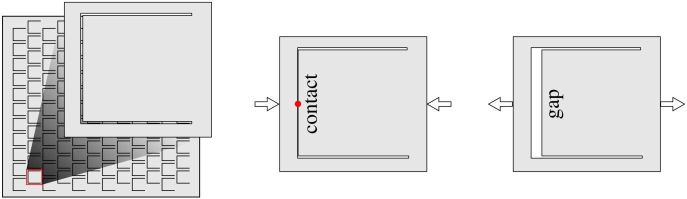

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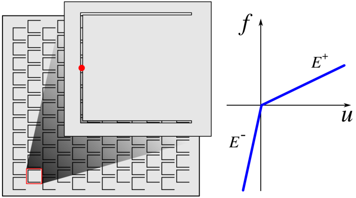

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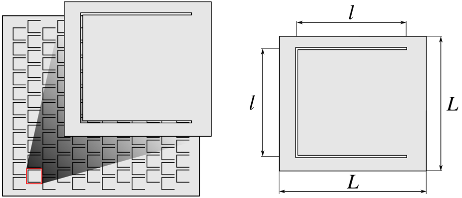

$$E^+/E^- \approx 1 - l/L$$

Contact-based architectured material

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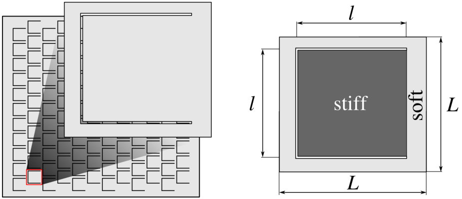
- Stiff in compression / soft in tension
- Use different materials



$$E^{+}/E^{-} \approx (1 - l/L)\, E^{\mathrm{soft}}/E^{\mathrm{stiff}}$$

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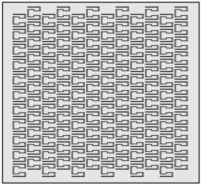

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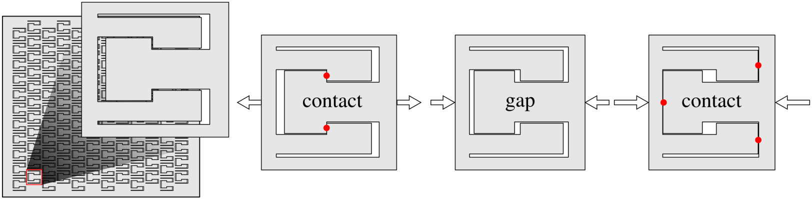

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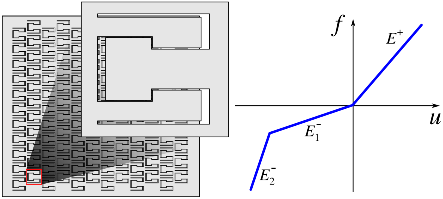

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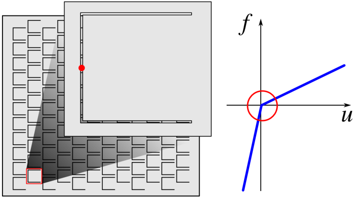
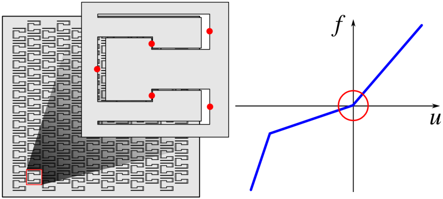
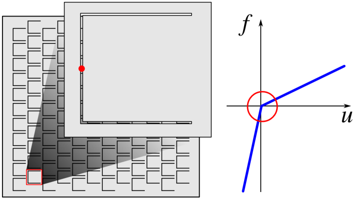
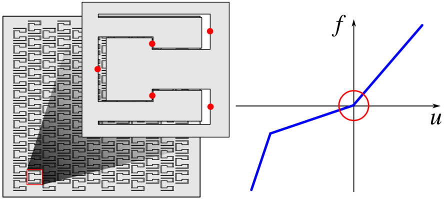




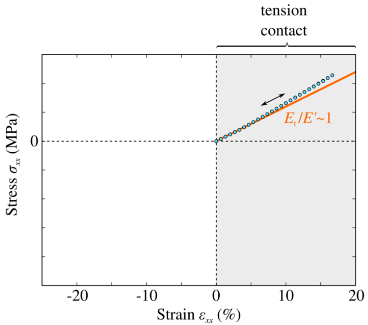
Contact-based architectured material

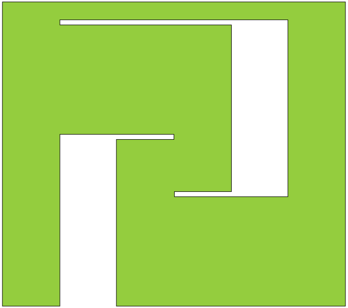
- Contact as a functional element
- Stiff in compression / soft in tension
- Use different materials
- Soft in compression/stiff in tension
- Extendable to 3D





■ Tension/compression test (FE simulation)





Governing equation for asymmetric materials

- Quastistatic 1D behavior:

$$\sigma = E(\nabla u + \alpha|\nabla u|) = \begin{cases} (1+\alpha)E\nabla u, & \text{if } \nabla u > 0, \text{ tension} \\ (1-\alpha)E\nabla u, & \text{if } \nabla u \le 0, \text{ compres.} \end{cases}, \quad -1 < \alpha < 1$$

- Elastic contrast γ:

$$\frac{E^+}{E^-} = \frac{1+\alpha}{1-\alpha} = \gamma$$

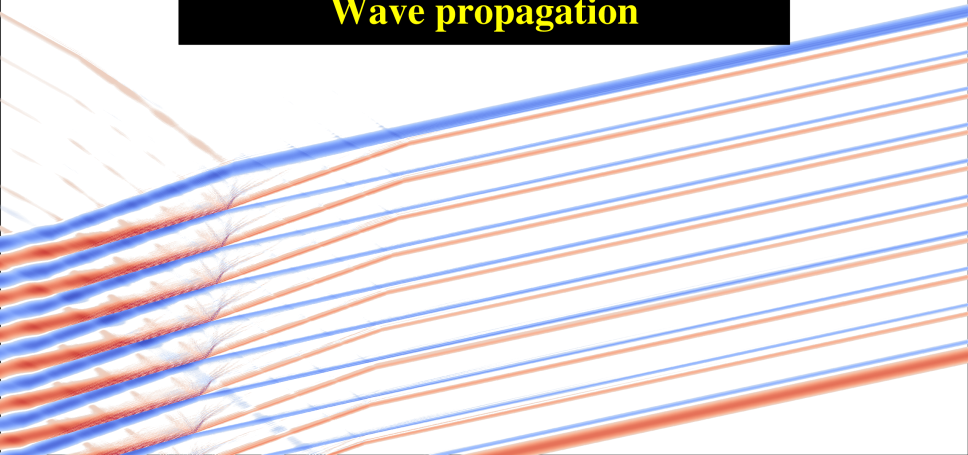
- Elastodynamic equation in 1D (the simplest approximation):

$$\rho\ddot{u} = E\nabla\left(\nabla u + \alpha|\nabla u|\right) + \underbrace{\mu\Delta\dot{u}}_{\text{damping}}$$

- Wave celerity c:

$$c = \begin{cases} \sqrt{(1+\alpha)E/\rho}, & \text{if } \nabla u > 0, \text{ tension} \\ \sqrt{(1-\alpha)E/\rho}, & \text{if } \nabla u \le 0, \text{ compres.} \end{cases}, \quad \frac{c^+}{c^-} = \sqrt{\frac{1+\alpha}{1-\alpha}} = \sqrt{\gamma}$$

Wave propagation



System set-up

- One dimensional wave propagation
- Single oscillation: $F \approx \sin(\omega_0 t) = \sin(2\pi c_0 t/\lambda_0)$
- Scale separation $\lambda_0 \gg l$, where l is the characteristic element size
- Elastic contrast $\alpha = 0.3 \quad \Rightarrow \quad \gamma = E^+/E^- \approx 1.86$

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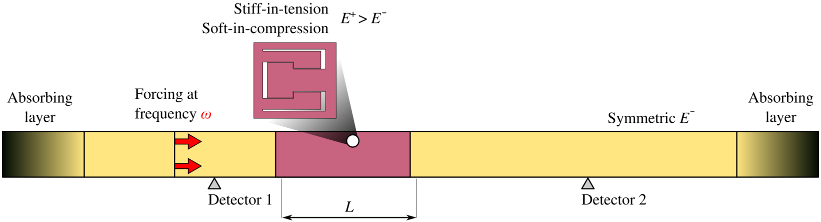

Configuration 1

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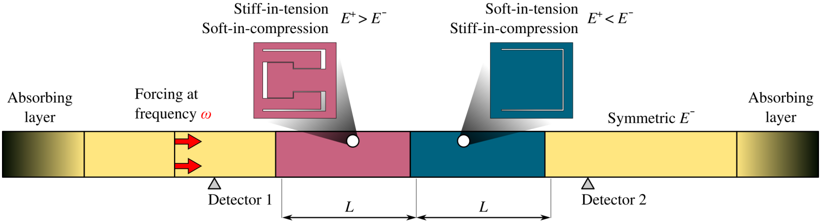

Configuration 2

System set-up

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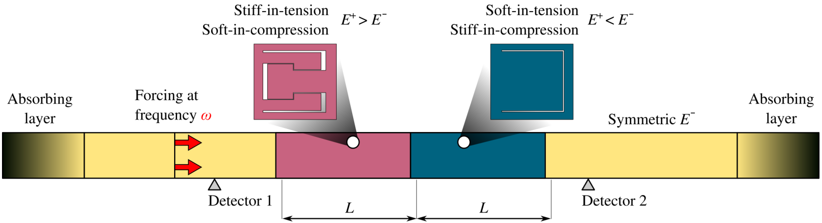

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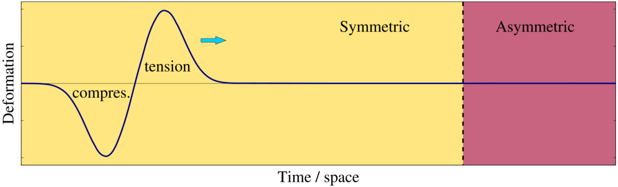

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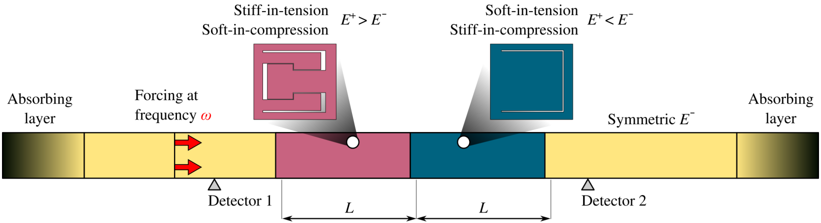

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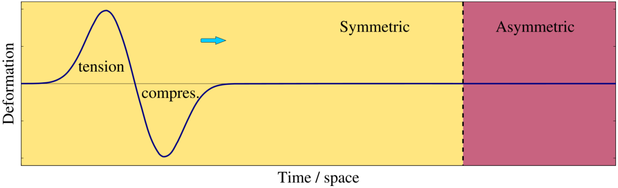
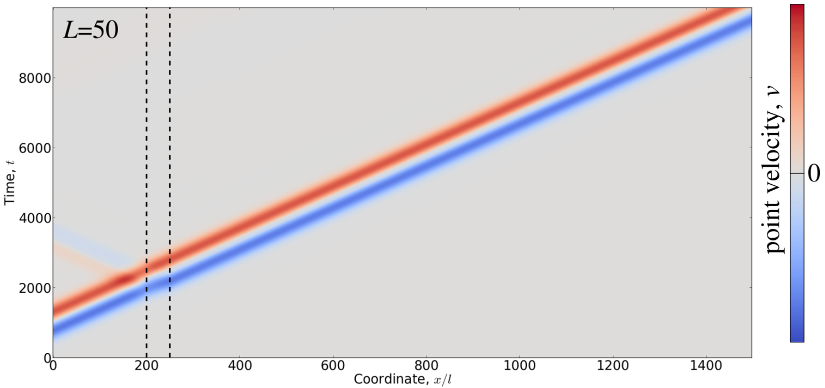


Effect of the length of the asymmetric section

Separation of wave-components

tensile (faster) part preceeds compressive one



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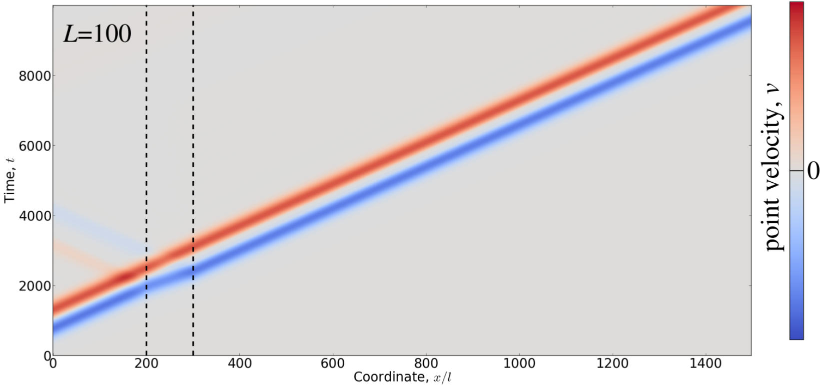

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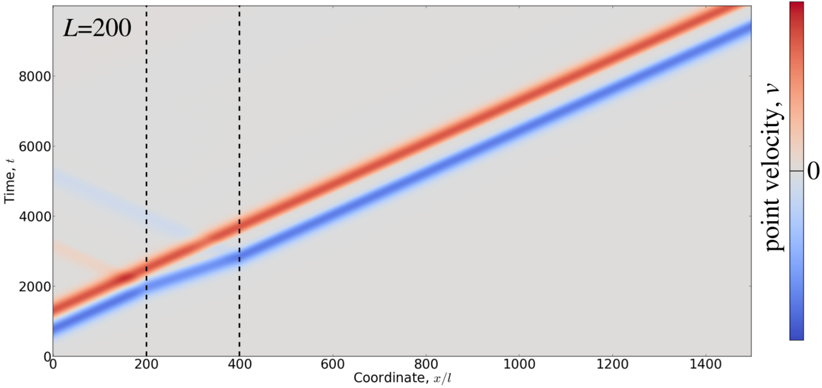

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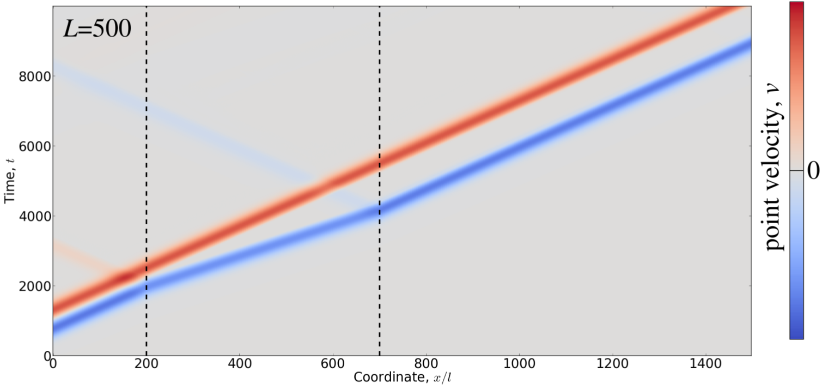

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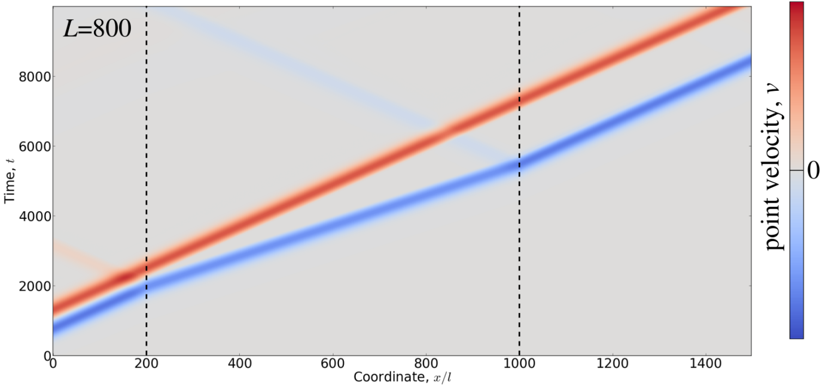

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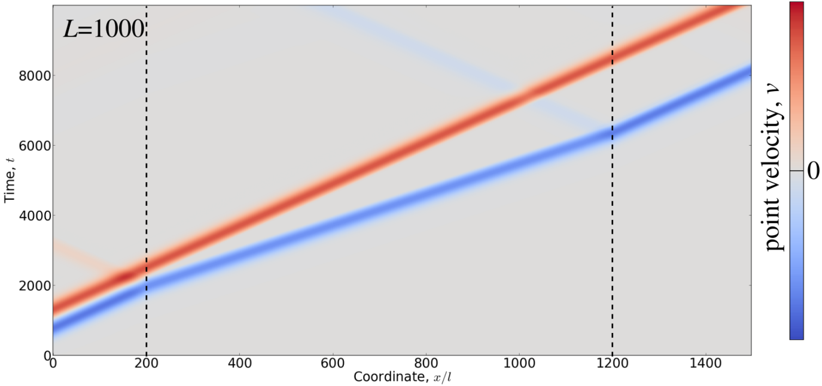

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Overlap of wave-components

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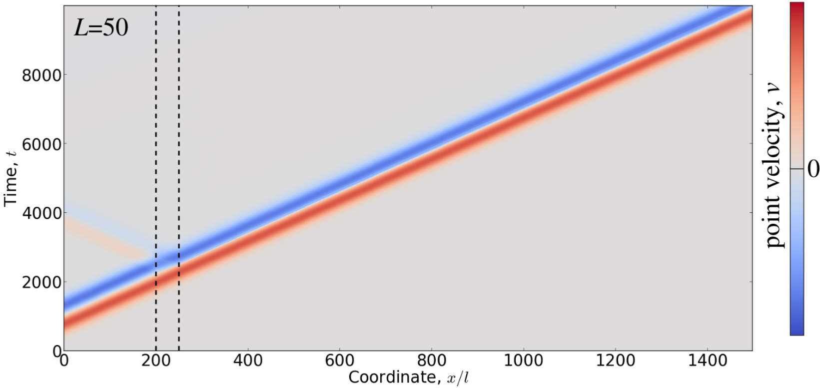

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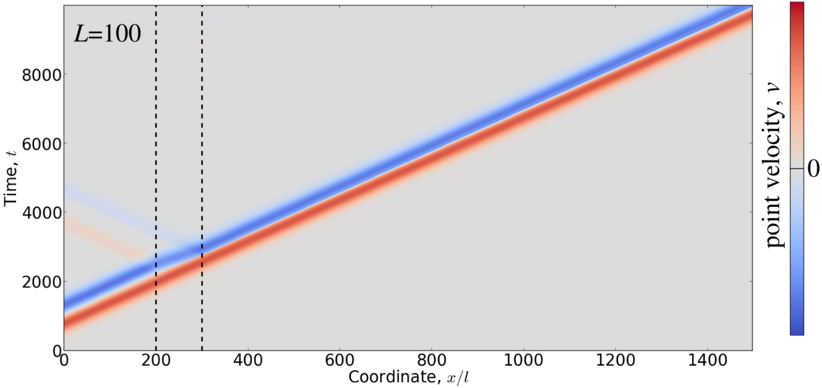

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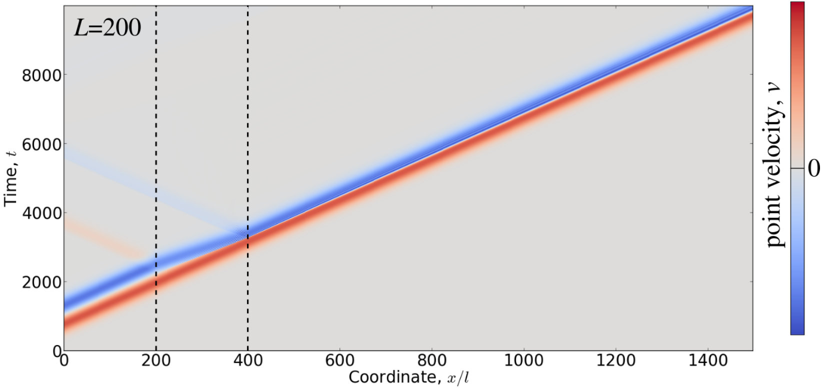

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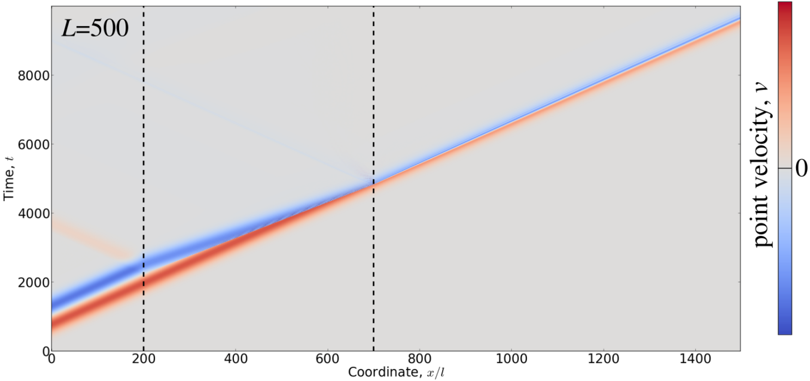

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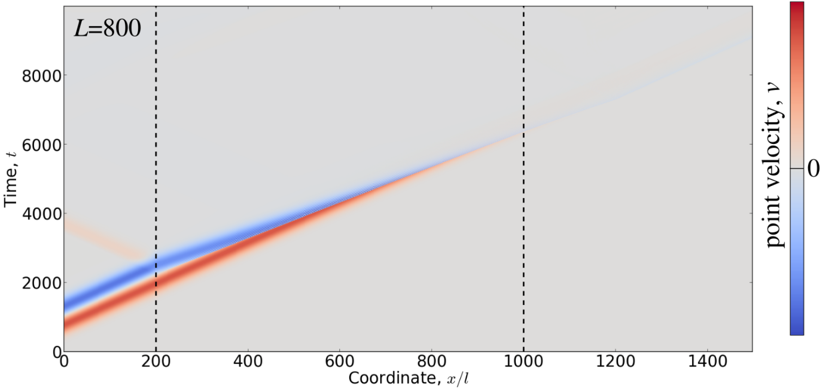

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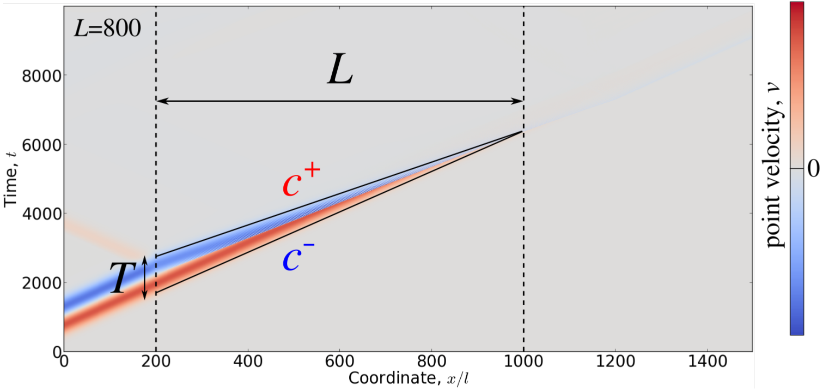
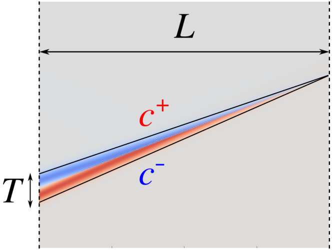


Effect of the length of the asymmetric section

Overlap of wave-components

- To let + and - waves fully overlap

$$T \le \frac{L}{c_l^-} - \frac{L}{c_l^+}$$



- Since $T = 2\pi/\omega_0$, $c^+ = \sqrt{(1+\alpha)E/\rho}$
- Then the full overlap occurs at length L_o

$$L_o \ge \frac{2\pi c^+}{\omega_0 \left[\frac{c^+}{c^-} - 1\right]} \Rightarrow \boxed{L_o \ge \frac{2\pi c^+}{\omega_0 \left(\sqrt{\gamma} - 1\right)}}$$

- In terms of wavelength:

$$\boxed{L_o \ge \frac{\lambda_0}{\sqrt{\gamma} - 1}} \text{ if } c^0 = c^+ \quad \text{or} \quad \boxed{L_o \ge \lambda_0 + \frac{\lambda_0}{\sqrt{\gamma} - 1}} \text{ if } c^0 = c^-$$

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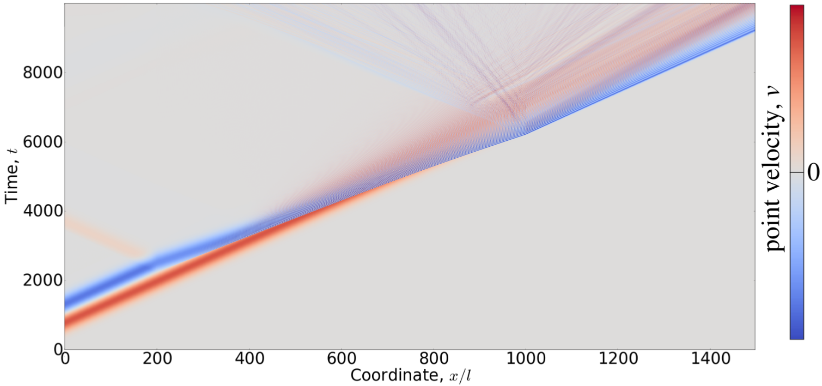
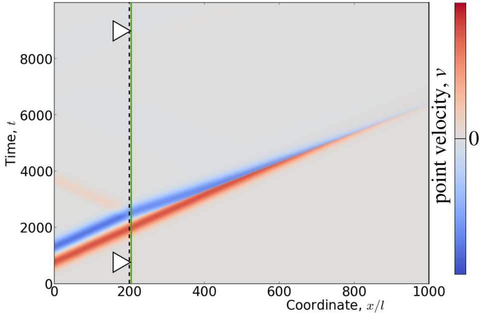
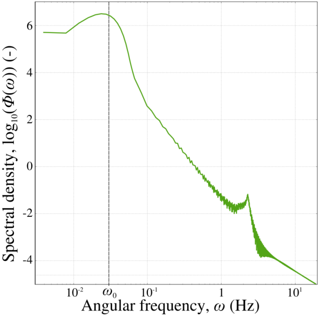


No dissipation, $\mu = 0$

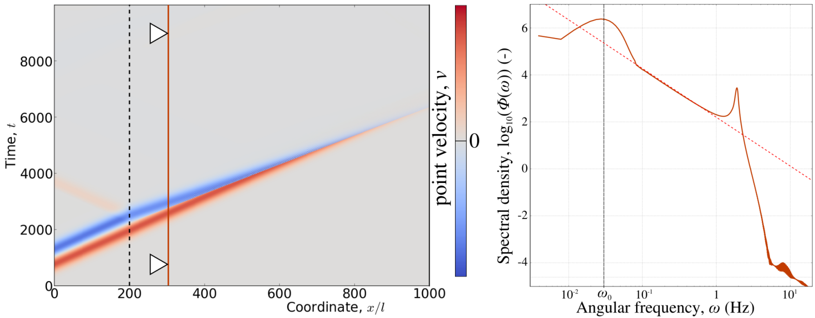
Spectrum of strain $\nabla u(t) \to \Phi(\omega)$ at given location x





Spectral analysis

Spectrum of strain $\nabla u(t) \to \Phi(\omega)$ at given location x



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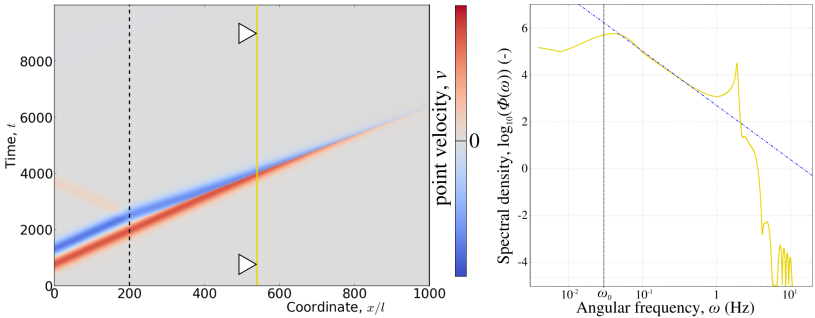

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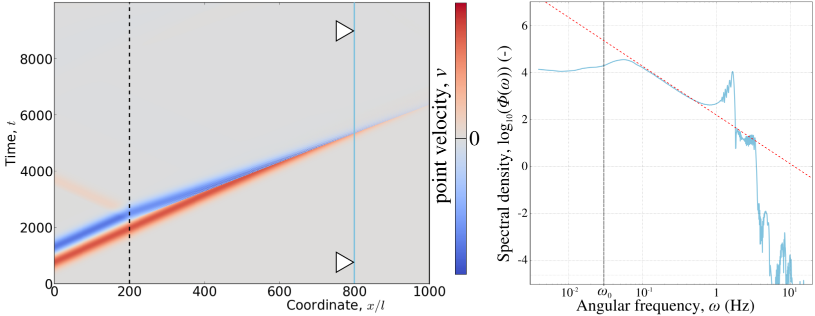

Spectral analysis

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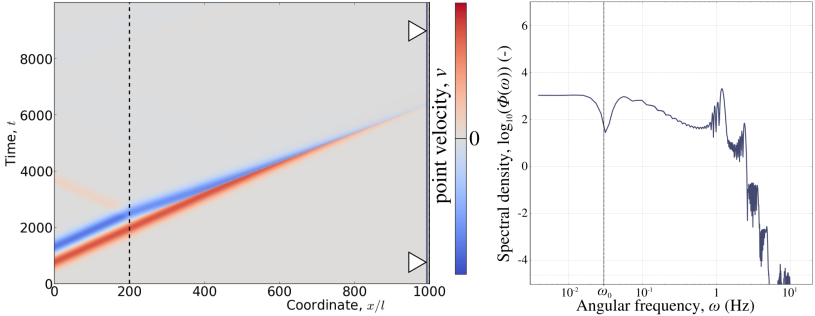
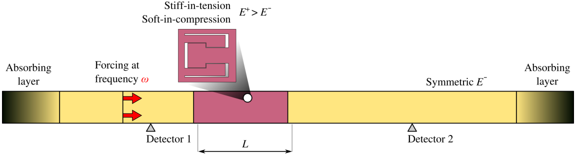


Energy migrates towards high-frequencies, where it is dissipated.

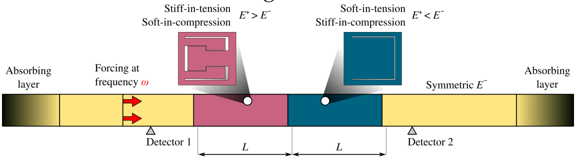
Parametric study

- Filtering properties
- Filtering factor: $\mathcal{F} = \log_{10}(\text{Energy out}/\text{Energy in})$
- Energy through detector 2:

$$E_{\text{out}} = \int_{t_1}^{t_2} [\dot{E}_k + \dot{E}_p] dt \qquad (1)$$

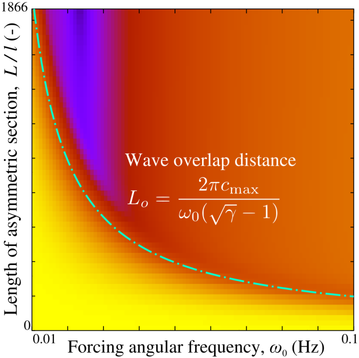


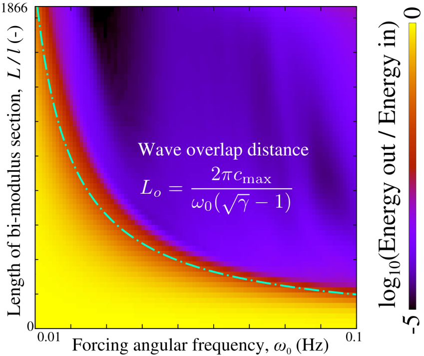
Configuration 1



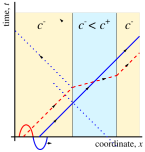
Configuration 2

Parametric study

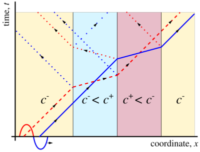




Configuration 1: single overlap



Configuration 2: double overlap



Conclusions and perspectives

Preliminary results

- Internal contact is a powerful non-linear mechanism in design of architectured materials
- Resulting elastic asymmetry can be used to overalp tensile and compressive wave components
- The system can be designed such that the full overlap occurs at

$$L_o = \frac{\lambda_0}{\sqrt{\gamma} - 1}$$

therefore, for high contrast γ, the damping system L can remain small.

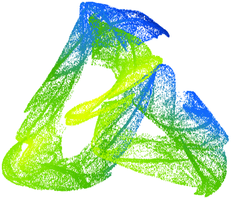
Perspectives

- Apply more advanced theories in 1D

  e.g., [1] Milton & Willis, On the modification of Newton's second law ..., PRSL A 463 (2007)
- Extension for 2D/3D fully anisotropic elastic asymmetry

Vibration of asymmetric materials with internal contacts

Examples of 4D strange attractor projected on 3D $(x_1, \dot{x}_1, x_2, \dot{x}_2)$



Thank you for your attention!

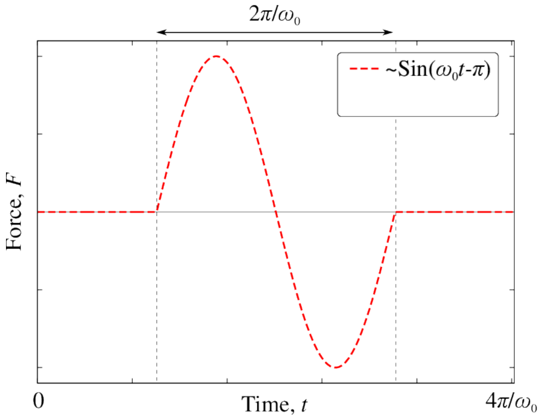
[A] Yastrebov, *"Wave-filtering properties of elastically-asymmetric architected materials"*, under review (2017).

arxiv.org/abs/1712.06294

Bonus: Smothed single oscillation

- Single oscillation: $F \approx \sin(\omega_0 t) = \sin(2\pi c_0 t/\lambda_0)$

$$F = \begin{cases} \mathrm{sign}(\omega_0)\left[e^{-(2|\omega_0|t/\pi-3)^2} - e^{-(2|\omega_0|t/\pi-5)^2}\right], & \text{if } 0 \le t \le 4\pi/|\omega_0| \\ 0, & \text{otherwise.} \end{cases}$$



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