Slip propagation at interfaces with a non-uniform contact pressure distribution

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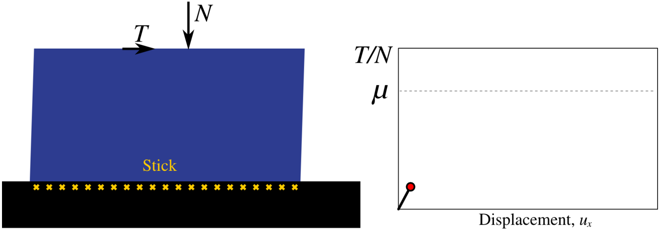
Outline

- Introduction
- Parabolic pressure valley
- Localized pressure valley
- Conclusions

Introduction

Sliding onset

- Simplified/naive vision of friction



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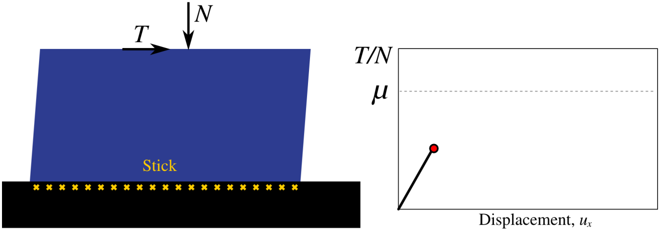

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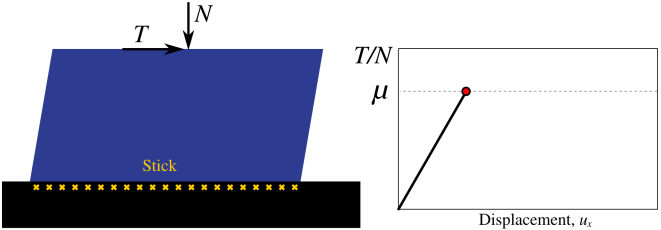

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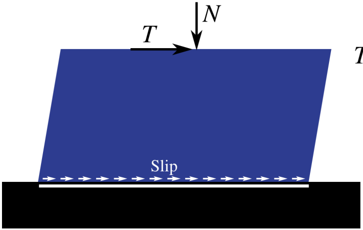

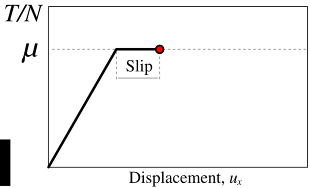

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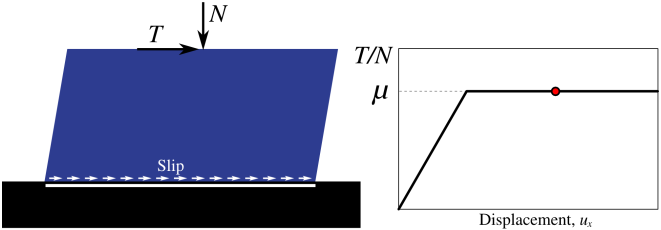

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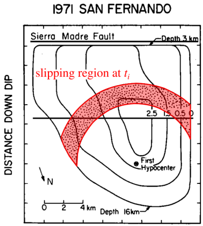
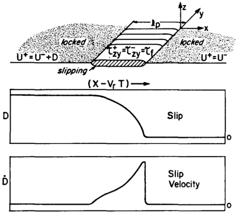
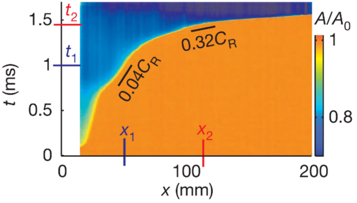
- Simplified/naive vision of friction
- As we know, in reality is not that simple[1,2]







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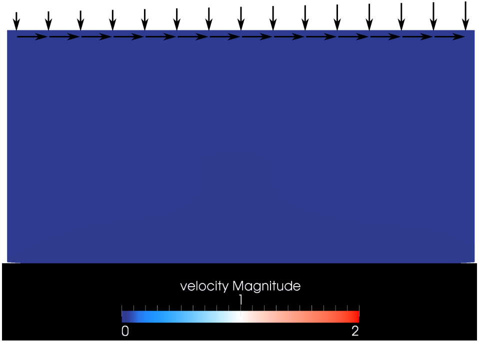

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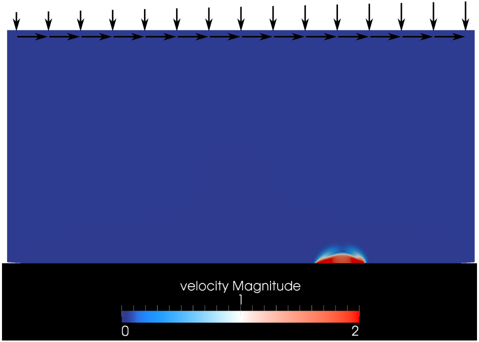

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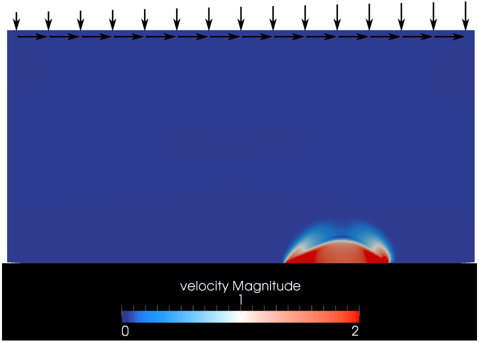

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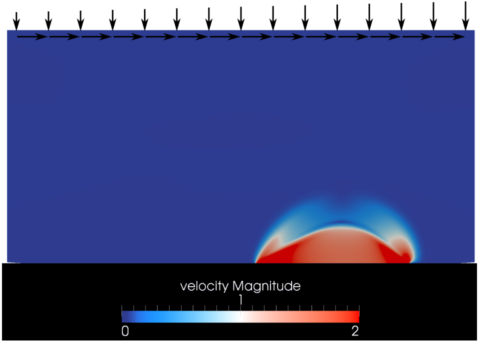

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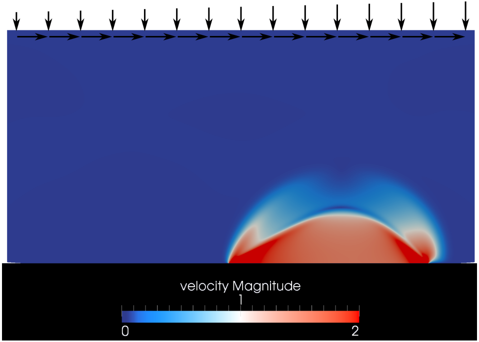

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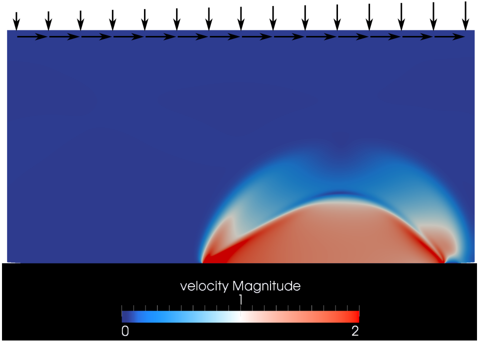

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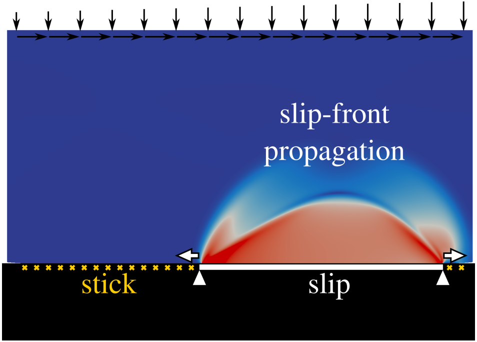

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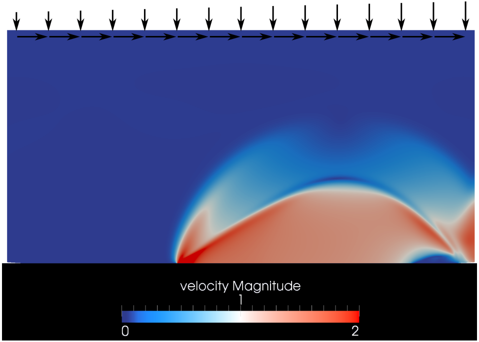

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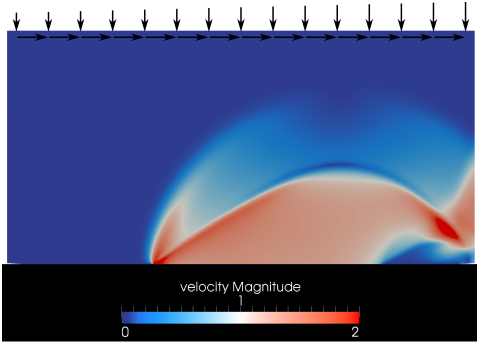

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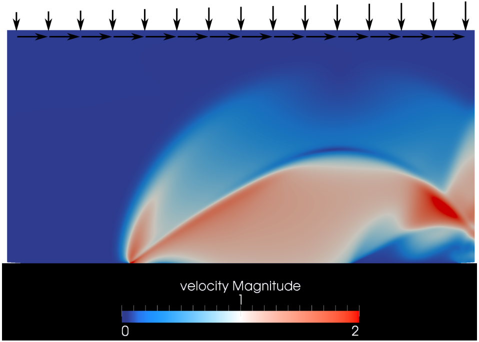

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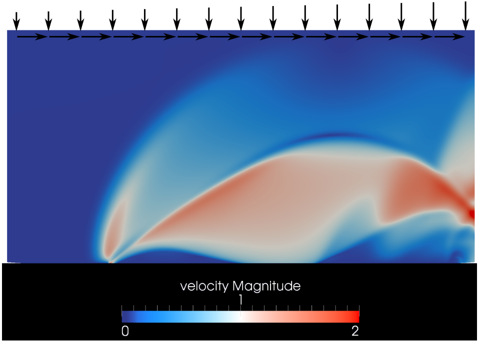

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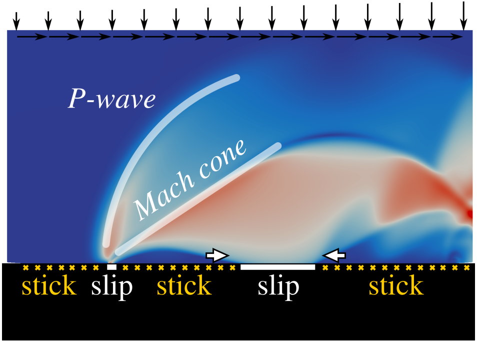

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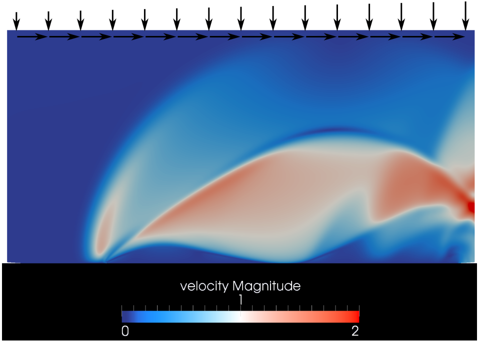

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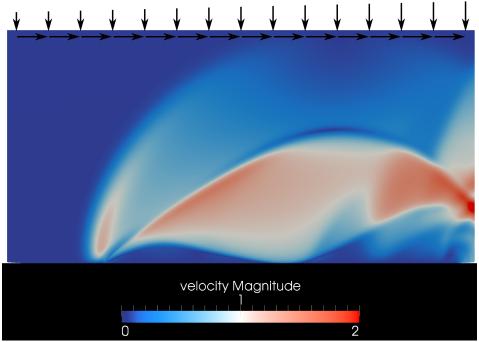
- Even if the stress is uniform at the interface[4-6]



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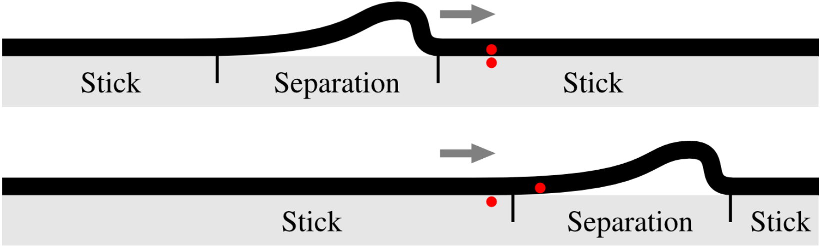
Emergence of an opening wave in Coulomb’s friction law at the interface between an elastic layer and a rigid flat[1]



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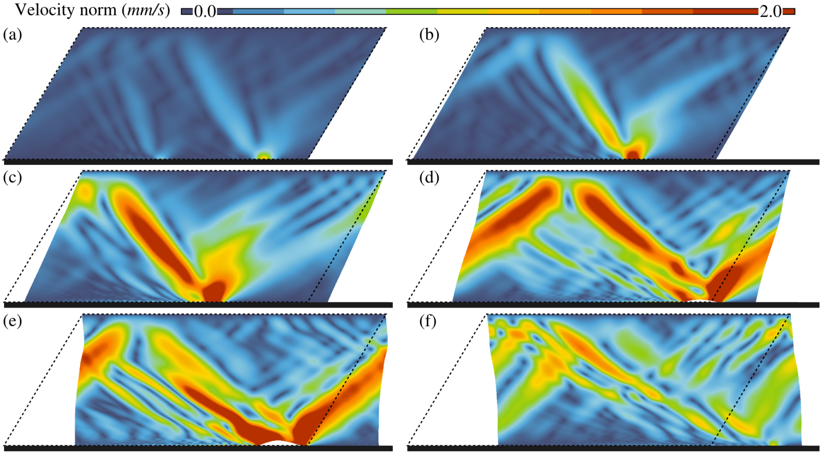

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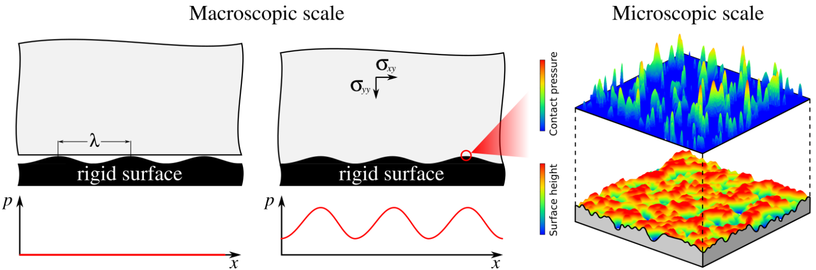
Non-uniform contact pressure

- Stability of the sliding depends on the local contact pressure distribution[1] $p(x,y)$
- **Macroscopic scale:**
  pressure depends on the shape of contacting solids, on loads, pore pressure[1-3] and material behaviour
- **Microscopic scale:**
  pressure depends on roughness and on material behaviour



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[1] More generally, on interface tractions.

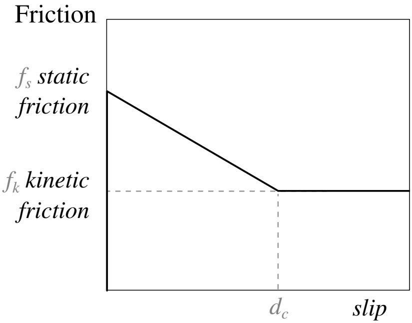
Frictional law

- Slip weakening friction law

$$f(d) = \begin{cases} f_s - \frac{(f_s - f_k)}{d_c} d, & \text{if } d \le d_c, \\ f_k, & \text{otherwise,} \end{cases}$$

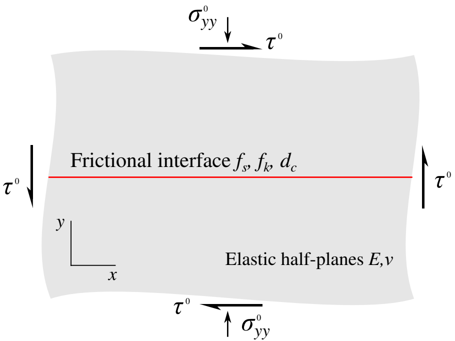
where f_s, f_k are the static and kinetic coefficients of friction, respectively, and d_c is the characteristic slip length.

- Simplified form of a more general rate-and-state friction

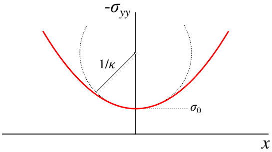


Problem set-up

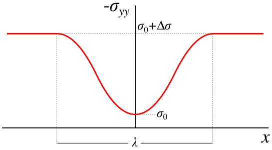
- Pressurized σ_{yy}^0 and sheared τ^0 interface between two similar[2] elastic E, ν half-planes
- Plane strain formulation



Case 1: parabolic pressure profile (κ,σ_0)



Case 2: localized pressure valley (λ,σ_0,$\Delta\sigma$)



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Analogy friction-fracture

- **D.J. Andrews** (1976)
  - ⋆ *Rupture Propagation With Finite Stress in Antiplane Strain*, J. Geophys. Res., 81
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  - ⋆ *Frictional sliding modes along an interface between identical . . .* , JMPS, 53
- **Jay Fineberg et al.** (2004, 2010, 2014)
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  - ⋆ O. Ben-David, G. Cohen, J. Fineberg (2010), *The dynamics of the onset of frictional slip*, Science 330
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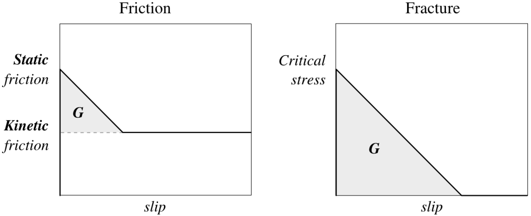

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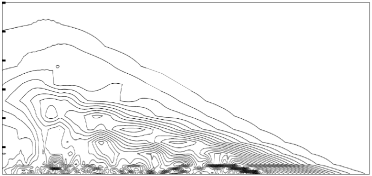
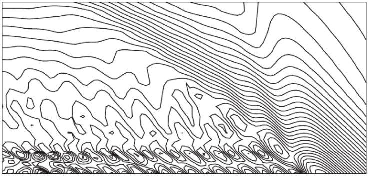


Fracture, *Coker et al.* (2003)



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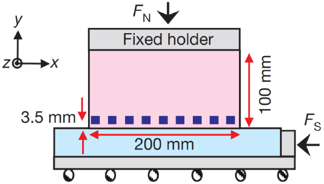
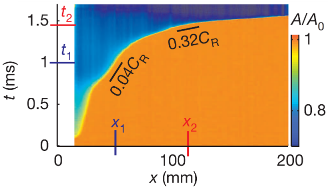
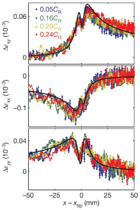
- **Jay Fineberg et al.** (2004, 2010, 2014)
  - ⋆ S.M. Rubinstein, G. Cohen, J. Fineberg (2004), *Detachment fronts and the onset of dynamic friction*, Nature 430
  - ⋆ O. Ben-David, G. Cohen, J. Fineberg (2010), *The dynamics of the onset of frictional slip*, Science 330
  - ⋆ I. Svetlizky, J. Fineberg (May 2014), *Classical shear cracks drive the onset of dry frictional motion*, Nature 509







Slip-stability analysis

- Consider situation that slip-weakening zone is localized near slip edges (small scale yielding[1] assumption)

$$u_{slip}(x) > d_c \text{ in } x \in (-a+\epsilon, a-\epsilon), \quad \epsilon \ll a$$

otherwise, see[2,3]

- Critical energy release rate

$$G_c(x) = \frac{1}{2} d_c (f_s - f_k) |\sigma_{yy}(x)|$$

- Energy release rate

$$G(a) = \frac{(1-\nu^2) K_{II}^2(a)}{E}$$

[1] Palmer & Rice, *The growth of slip surfaces in the progressive failure of over-consolidated clay*, 1973, Proc. Royal Soc. London 332
[2] Campillo & Ionescu, *Initiation of antiplane shear instability under slip dependent friction*, 1997, J. Geophys. Res. 102
[3] Uenishi & Rice, *Universal nucleation length for slip-weakening rupture instability under nonuniform fault loading*, 2003, J. Geophys. Res. 108

Slip-stability analysis II

- Stress intensity factor[1]:

$$K_{II}(a) = \underbrace{\tau_0\sqrt{\pi a}}_{\text{ambient shear}} \underbrace{- \frac{1}{\sqrt{\pi a}}\int\limits_{-a}^{a} f_k \sigma_{yy}(x) \frac{\sqrt{a+x}}{\sqrt{a-x}} dx}_{\text{residual frictional shear}}$$

- Introduce function $\mathcal{F}$:

$$\mathcal{F} = G(a) - G_c(a) : \quad \begin{cases} \mathcal{F} > 0, \text{ unstable slip,} \\ \mathcal{F} < 0, \text{ no spontaneous slip} \end{cases}$$

- Search stable $\dfrac{\partial \mathcal{F}}{\partial a} < 0$ and unstable $\dfrac{\partial \mathcal{F}}{\partial a} > 0$ roots of $\mathcal{F} = 0$

[1] Erdogan, F., 1962. *On the stress distribution in plates with collinear cuts under arbitrary loads*, Proc. 4th US Nat. Cong. Appl. Mech. 1.

Model parameters

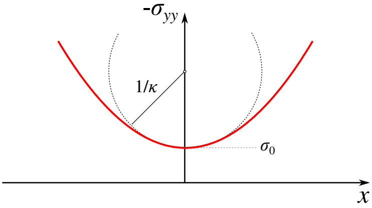
- Isotropic homogeneous half-spaces in plane strain (typical granite)[1,2]:
  Shear modulus $\mu = 30$ GPa, Poisson's ratio $\nu = 0.25$
- Friction parameters:
  - static friction $f_s = 0.8$
  - kinetic friction $f_k = 0.6$
  - weakening distance $d_c = 450\ \mu$m
- Valley pressure:
  $\sigma_0 \in [10, 300]$ MPa as pressure ≈ 28 MPa/km $\times$ depth

[1] Rummel, F., Alheid, H., Frohn, C., 1978. Dilatancy and fracture induced velocity changes in rock and their relation to frictional sliding. In: Rock Friction and Earthquake Prediction
[2] Uenishi, K., Rice, J. R., 2003. Universal nucleation length for slip-weakening rupture instability under nonuniform fault loading. Journal of Geophysical Research: Solid Earth

Parabolic pressure valley

Stability equation

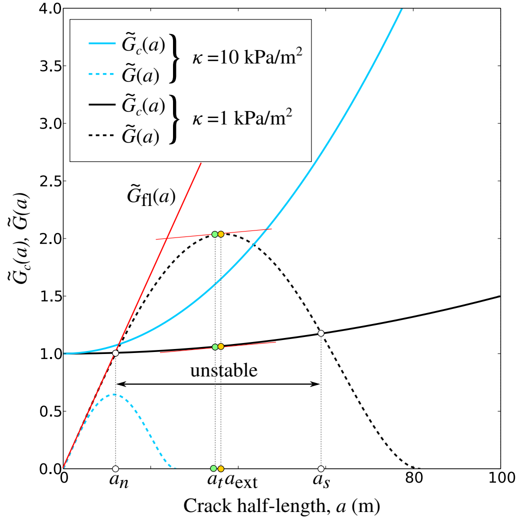
Case 1: parabolic pressure profile (κ,σ_0)



- Pressure distribution: $\sigma_{yy} = \sigma_0 + \dfrac{\kappa x^2}{2}$
- Notations $\tau_0 = f_0\sigma_0$, $\tau_k = f_k\sigma_0$, $\tau_s = f_s\sigma_0$
- Stress intensity factor: $K_{II} = (\tau_0 - f_k\sigma_0 - f_k\kappa a^2/4)\sqrt{\pi a}$
  $K_{II} > 0 \quad \Leftrightarrow \quad a \le 2\sqrt{(\tau_0 - \tau_k)/f_k\kappa}$.
- Consider a normalized yield function:

$$\frac{2\mathcal{F}}{d_c\sigma_0(f_s - f_k)} = \tilde{G}(a) - \tilde{G}_c(a)$$

Graphical representation



Note, $\tilde{G}_{\text{fl}}$ corresponds to a frictionless case.

Resulting equation and asymptotics

- Search for roots of:

$$\tilde{G}(a) - \tilde{G}_c(a) = \frac{2\pi a(1-\nu^2)[\tau_0 - f_k(\sigma_0 + \kappa a^2/4)]^2}{E d_c(\tau_s - \tau_k)} - \left(1 + \frac{\kappa a^2}{2\sigma_0}\right) = 0$$

- In the limit of a flat stress distribution $\kappa \to 0$, we obtain a unique solution:

$$a_n = \frac{\mu d_c(\tau_s - \tau_k)}{\pi(1-\nu)(\tau_0 - \tau_k)^2},$$

which is nothing but Andrews' critical nucleation length[1].

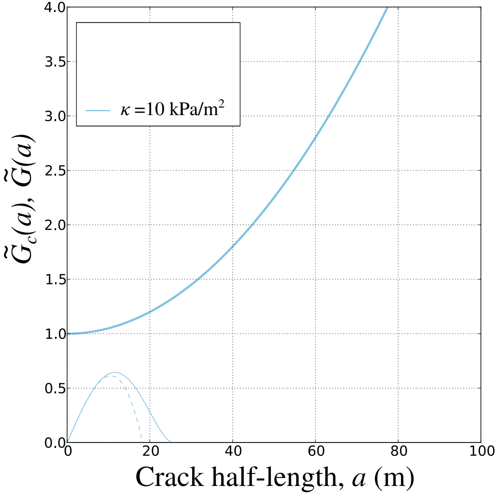
- If $\tau_0 = \tau_s$, this nucleation length reduces to

$$a_n\Big|_{\tau_0=\tau_s} \approx \frac{0.3183\mu d_c}{(1-\nu)(\tau_s - \tau_k)} < \underbrace{\frac{0.579\mu d_c}{(1-\nu)(\tau_s - \tau_k)}}_{\text{from}^{[2]}}$$

[1] J.D. Andrews, 1976. *Rupture Propagation With Finite Stress in Antiplane Strain*, J. Geophys. Res., 81
[2] Uenishi & Rice, 2003. *Universal nucleation length for slip-weakening rupture instability under nonuniform fault loading*, J. Geophys. Res. 108

Some examples

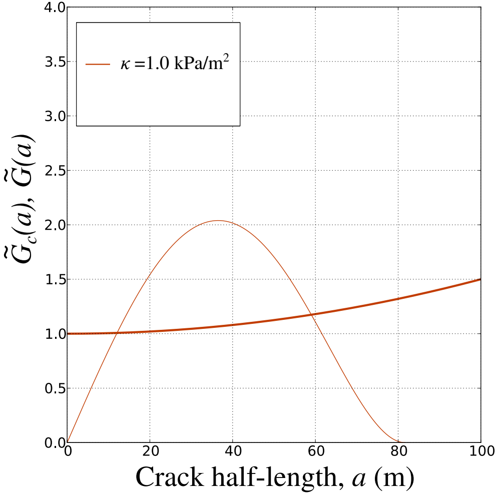
- Normalized energy release rate $\tilde{G}(a)$ and its critical value $\tilde{G}_c(a)$



$\sigma_0 = 10$ MPa, $f_0 = 0.7$

Some examples

- Normalized energy release rate $\tilde{G}(a)$ and its critical value $\tilde{G}_c(a)$



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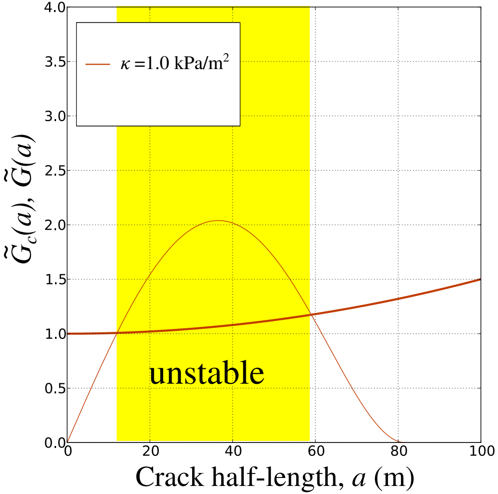
Some examples

- Normalized energy release rate $\tilde{G}(a)$ and its critical value $\tilde{G}_c(a)$



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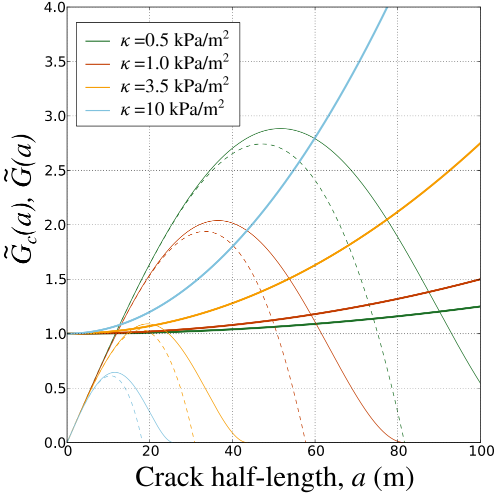
Some examples

- Normalized energy release rate $\tilde{G}(a)$ and its critical value $\tilde{G}_c(a)$



$\sigma_0 = 10$ MPa, $f_0 = 0.7$

Approximate analysis: possibility of unstable slip?

- Neglect fifth order term:

$$\frac{2\mathcal{F}}{d_c(\tau_s-\tau_k)} \approx A_1\tilde{a} - A_2\tilde{a}^2 - A_3\tilde{a}^3 - 1 = 0$$

where $\tilde{a} = a/d_c$ and

$$A_3 = \frac{\pi(1-\nu)f_k\kappa d_c^2(\tau_0-\tau_k)}{2\mu(\tau_s-\tau_k)},\; A_2 = \frac{\kappa d_c^2}{2\sigma_0},\; A_1 = \frac{\pi(1-\nu)(\tau_0-\tau_k)^2}{\mu(\tau_s-\tau_k)}$$

- Search for $\partial\mathcal{F}/\partial a = 0$, we obtain $A_1 - 2A_2\tilde{a} - 3A_3\tilde{a}^2 = 0$

$$\tilde{a}_t = \frac{\mu(\tau_s-\tau_k)}{3\pi f_k\sigma_0(1-\nu)(\tau_0-\tau_k)}\underbrace{\left[\sqrt{1+\frac{6\pi^2 f_k(1-\nu)^2(\tau_0-\tau_k)^3}{\mu^2(f_s-f_k)^2\kappa d_c^2}}-1\right]}_{\text{Normally, it is } \gg 1}.$$

Approximate analysis: possibility of unstable slip?

- Neglect fifth order term:

$$\frac{2\mathcal{F}}{d_c(\tau_s - \tau_k)} \approx A_1\tilde{a} - A_2\tilde{a}^2 - A_3\tilde{a}^3 - 1 = 0$$

where $\tilde{a} = a/d_c$ and

$$A_3 = \frac{\pi(1-\nu)f_k\kappa d_c^2(\tau_0 - \tau_k)}{2\mu(\tau_s - \tau_k)},\ A_2 = \frac{\kappa d_c^2}{2\sigma_0},\ A_1 = \frac{\pi(1-\nu)(\tau_0 - \tau_k)^2}{\mu(\tau_s - \tau_k)}$$

- Search for $\partial\mathcal{F}/\partial a = 0$, we obtain $A_1 - 2A_2\tilde{a} - 3A_3\tilde{a}^2 = 0$

$$a_t \approx \sqrt{\frac{2(\tau_0 - \tau_k)}{3\kappa f_k}}$$

Approximate analysis: possibility of unstable slip?

- Neglect fifth order term:

$$\frac{2\mathcal{F}}{d_c(\tau_s - \tau_k)} \approx A_1\tilde{a} - A_2\tilde{a}^2 - A_3\tilde{a}^3 - 1 = 0$$

where $\tilde{a} = a/d_c$ and

$$A_3 = \frac{\pi(1-\nu)f_k\kappa d_c^2(\tau_0 - \tau_k)}{2\mu(\tau_s - \tau_k)},\; A_2 = \frac{\kappa d_c^2}{2\sigma_0},\; A_1 = \frac{\pi(1-\nu)(\tau_0 - \tau_k)^2}{\mu(\tau_s - \tau_k)}$$

- Search for $\partial\mathcal{F}/\partial a = 0$, we obtain $A_1 - 2A_2\tilde{a} - 3A_3\tilde{a}^2 = 0$

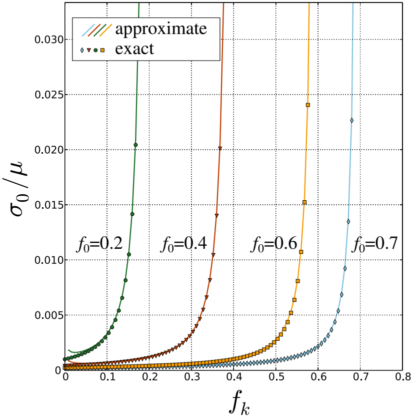
$$a_t \approx \sqrt{\frac{2(\tau_0 - \tau_k)}{3\kappa f_k}}$$

- Then the unstable slip ($\mathcal{F}(a_t) > 0$) is possible if:

$$\boxed{\frac{\sigma_0}{\mu} \gtrapprox \left[\frac{3(f_s - f_k)^2}{8\pi^2 f_k} \cdot \frac{\kappa d_c^2}{(1-\nu)^2\mu} \cdot \frac{(2f_k + f_0)^2}{(f_0 - f_k)^5}\right]^{1/3}}$$

Approximate analysis: possibility of unstable slip?

- Minimal pressure σ_0 needed to have unstable slip



For average granite properties[1]:
Poisson's ratio $\nu = 0.25$, shear modulus $\mu = 30$ GPa,
friction $f_s = 0.8$, $f_k = 0.6$, and $d_c = 450\ \mu$m

[1] Uenishi & Rice, *Universal nucleation length for slip-weakening rupture instability under nonuniform fault loading*, 2003, J. Geophys. Res. 108

Approximate analysis II: arrest length

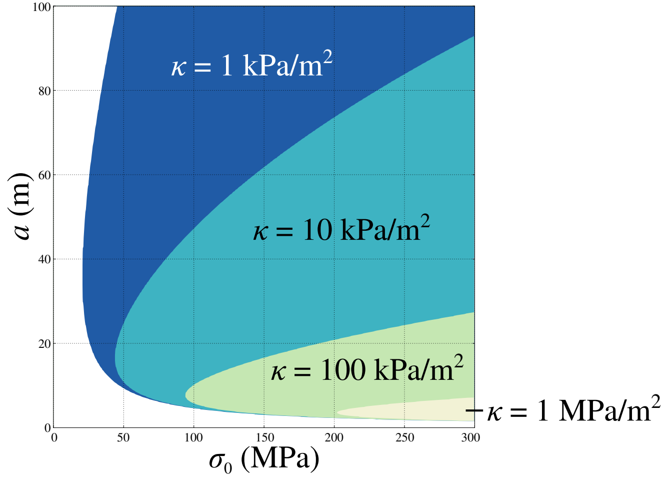
- Maximum of energy release rate $\partial G/\partial a = 0$:

$$a_{\text{ext}} = 2\sqrt{\frac{\sigma_0(f_0 - f_k)}{5\kappa f_k}}$$

- If we assume that this maximum is located on the mid-way between two roots: $a_{\text{ext}} \approx (a_n + a_s)/2$
- Then the arrest length is given by:

$$\boxed{a_s \approx 4\sqrt{\frac{\sigma_0(f_0 - f_k)}{5\kappa f_k}} - \frac{(f_s - f_k)d_c}{\pi(1-\nu)(f_0 - f_k)^2} \cdot \frac{\mu}{\sigma_0}}$$

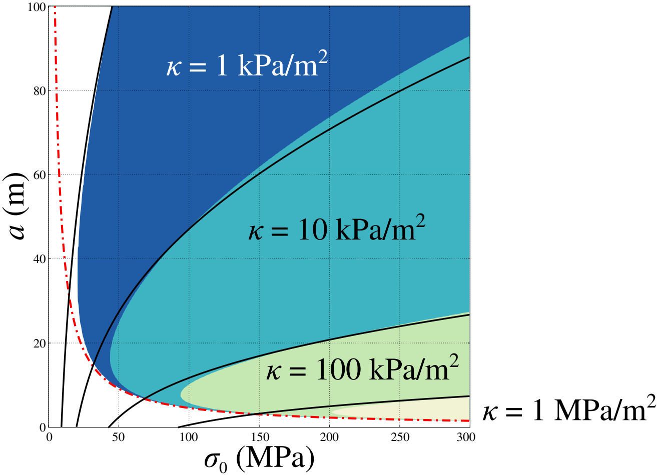
- Colours correspond to the region of unstable slip for different pressure curvatures κ



For average granite properties and $f_0 = 0.65$

Approximate analysis II: arrest length

- Colours correspond to the region of unstable slip for different pressure curvatures κ



For average granite properties and $f_0 = 0.65$

Intermediate conclusions

- For a pressure valley: $\sigma_{yy} = \sigma_0 + \dfrac{\kappa x^2}{2}$
- Unstable slip is possible if:

$$\frac{\sigma_0}{\mu} \gtrapprox \left[\frac{3(f_s - f_k)^2}{8\pi^2 f_k} \cdot \frac{\kappa d_c^2}{(1-\nu)^2\mu} \cdot \frac{(2f_k + f_0)^2}{(f_0 - f_k)^5}\right]^{1/3}$$

- Unstable slip starts at:

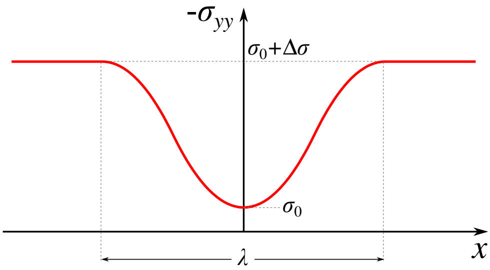
$$a_n \approx \frac{\mu d_c(\tau_s - \tau_k)}{\pi(1-\nu)(\tau_0 - \tau_k)^2}$$

- Unstable slip arrests at:

$$a_s \approx 4\sqrt{\frac{\sigma_0(f_0 - f_k)}{5\kappa f_k}} - \frac{(f_s - f_k)d_c}{\pi(1-\nu)(f_0 - f_k)^2} \cdot \frac{\mu}{\sigma_0}$$

Localized pressure valley

Case 2: localized pressure valley (λ,σ_0,$\Delta\sigma$)



- Pressure distribution:

$$\sigma(x) = \sigma_0 + \begin{cases} 8\Delta\sigma\left(\frac{x}{\lambda}\right)^2 & \text{, if } |x| \le \lambda/4 \\ \Delta\sigma\left(\frac{8|x|}{\lambda} - \frac{8x^2}{\lambda^2} - 1\right) & \text{, if } \lambda/4 < |x| \le \lambda/2 \\ \Delta\sigma & \text{, if } |x| > \lambda/2, \end{cases}$$

- As previously:

$$K_{II}(a) = \tau_0\sqrt{\pi a} - \frac{1}{\sqrt{\pi a}}\int\limits_{-a}^{a} f_k \sigma_{yy}(x)\frac{\sqrt{a+x}}{\sqrt{a-x}}dx$$

- Then

$$K_{II}(a) = \begin{cases} \left\langle \sqrt{\pi a}\tau_0 - J_1(a,a)\right\rangle, & \text{if } a < \lambda/4, \\ \left\langle \sqrt{\pi a}\tau_0 - J_1(\lambda/4,a) - J_2(a,a)\right\rangle, & \text{if } \lambda/4 \le a < \lambda/2, \\ \left\langle \sqrt{\pi a}\tau_0 - J_1(\lambda/4,a) - J_2(\lambda/2,a) - J_3(a,a)\right\rangle, & \text{if } a \ge \lambda/2, \end{cases}$$

where $\langle x\rangle = \max\{0, x\}$

$$J_1(l,a) = \frac{f_k}{\sqrt{\pi a}}\int\limits_{-l}^{l}\left[\sigma_0 + 8\Delta\sigma\left(\frac{x}{\lambda}\right)^2\right]\frac{\sqrt{a+x}}{\sqrt{a-x}}\,dx,$$

$$J_2(l,a) = \frac{f_k}{\sqrt{\pi a}}\left\{\int\limits_{-l}^{-\lambda/4} F_2(x,a)dx + \int\limits_{\lambda/4}^{l} F_2(x,a)dx\right\}, \text{ for } \lambda/4 \le l < \lambda/2,$$

where $F_2(x,a) = \left[(\sigma_0 - \Delta\sigma) + \frac{8\Delta\sigma}{\lambda}\left(|x| - \frac{x^2}{\lambda}\right)\right]\frac{\sqrt{a+x}}{\sqrt{a-x}}$,

$$J_3(l,a) = \frac{f_k}{\sqrt{\pi a}}\left\{\int\limits_{-l}^{-\lambda/2} F_3(x,a)dx + \int\limits_{\lambda/2}^{l} F_3(x,a)dx\right\}, \text{ for } l \ge \lambda/2, \text{ with } F_3 = (\sigma_0 + \Delta\sigma)\frac{\sqrt{a+x}}{\sqrt{a-x}}.$$

Stress intensity factor II

$$J_1(a,a) = f_k\sqrt{\pi a}\left(\sigma_0 + 4\Delta\sigma a^2/\lambda^2\right),$$

$$J_1(\lambda/4,a) = 2f_k\sqrt{\frac{a}{\pi}}\left[\left(\sigma_0 + \frac{4\Delta\sigma a^2}{\lambda^2}\right)\arcsin\left(\frac{\lambda}{4a}\right) - \frac{\Delta\sigma}{4\lambda}\sqrt{16a^2-\lambda^2}\right]$$

$$J_2(a,a) = 2f_k\sqrt{\frac{a}{\pi}}\left[\left(\sigma_0 - \Delta\sigma\left[1+4a^2/\lambda^2\right]\right)\left(\frac{\pi}{2} - \arcsin\frac{\lambda}{4a}\right) + \frac{7\Delta\sigma}{4\lambda}\sqrt{16a^2-\lambda^2}\right]$$

$$J_2(\lambda/2,a) = 2f_k\sqrt{\frac{a}{\pi}}\left[\left(\sigma_0 - \Delta\sigma\left[1+4a^2/\lambda^2\right]\right)\left(\arcsin\frac{\lambda}{2a} - \arcsin\frac{\lambda}{4a}\right) + \right.$$
$$\left. + \frac{\Delta\sigma}{4\lambda}\left(7\sqrt{16a^2-\lambda^2} - 12\sqrt{4a^2-\lambda^2}\right)\right]$$

$$J_3(a,a) = 2f_k\sqrt{\frac{a}{\pi}}(\sigma_0+\Delta\sigma)\left[\frac{\pi}{2} - \arcsin\frac{\lambda}{2a}\right]$$

Stability analysis

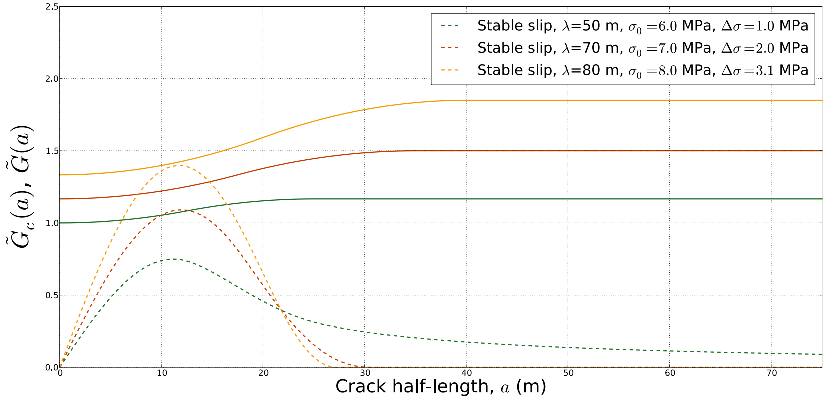
- Equating G and G_c gives:

$$\frac{2\mathcal{F}}{d_c\sigma_0(f_s-f_k)} = \tilde{G}(a) - \tilde{G}_c(a) = \frac{2(1-\nu^2)}{Ed_c\sigma_0(f_s-f_k)}K_{II}^2(a) - \frac{\sigma_{yy}(a)}{\sigma_0} = 0$$

  Only numerically solved...

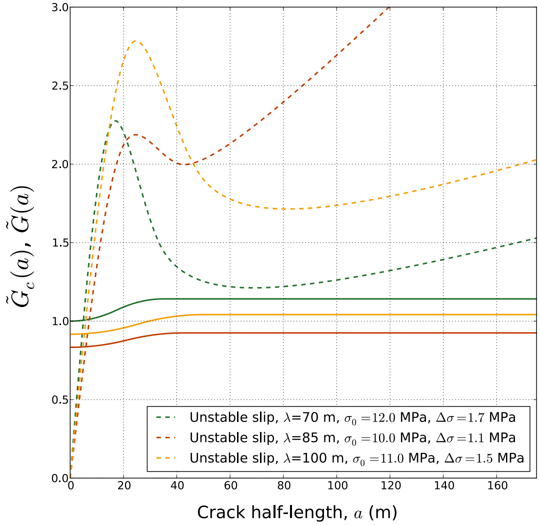
- Different scenarios:
  1. No roots
  2. One root
  3. Two roots
  4. Three roots

- Equation $G - G_c = 0$ does not have a solution



Scenario 2

- Equation $G - G_c = 0$ has one root (ultimately unstable)



Scenario 2

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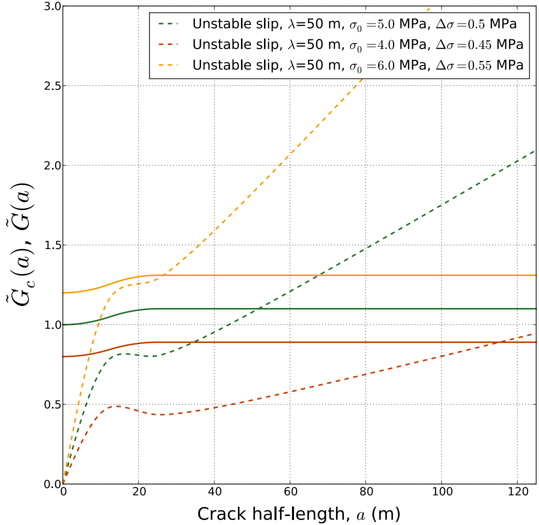
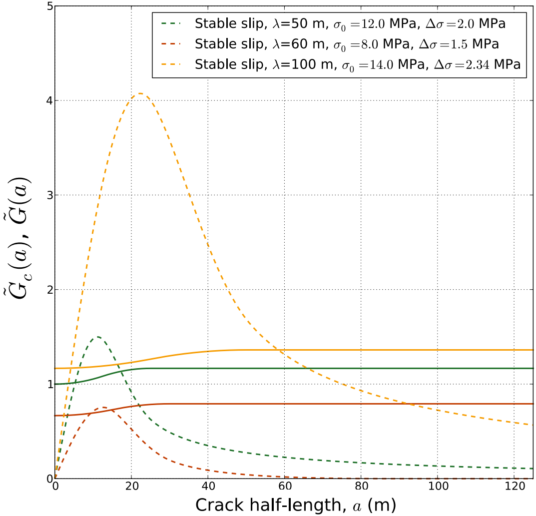


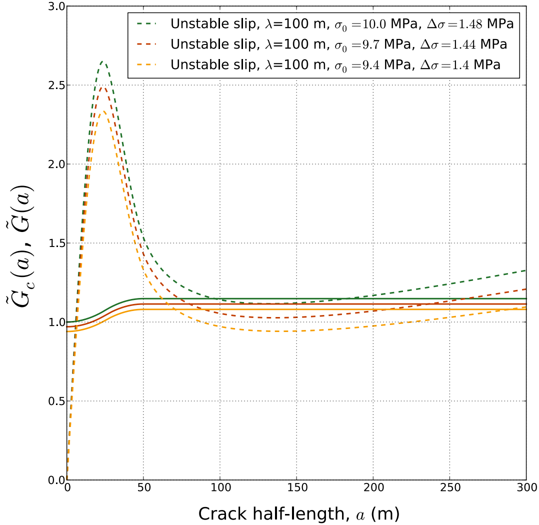
Scenario 3

- Equation $G - G_c = 0$ has two roots (ultimately stable)



Scenario 4

- Equation $G - G_c = 0$ has three roots (ultimately unstable)



First result

- Stress intensity factor

$$K_{II} = \sqrt{\pi a}\tau_0 - J_1(\lambda/4, a) - J_2(\lambda/2, a) - J_3(a, a)$$

- It can be shown that for $a/\lambda \gg 1$:

$$J_1(\lambda/4, a) \sim 1/\sqrt{a}, \qquad J_2(\lambda/2, a) \sim 1/\sqrt{a}$$

- Then for $a/\lambda \gg 1$:

$$K_{II} = \sqrt{\pi a}\,[f_0\sigma_0 - f_k(\sigma_0 + \Delta\sigma)]]$$

- Readily the condition for ultimately unstable slip can be derived:

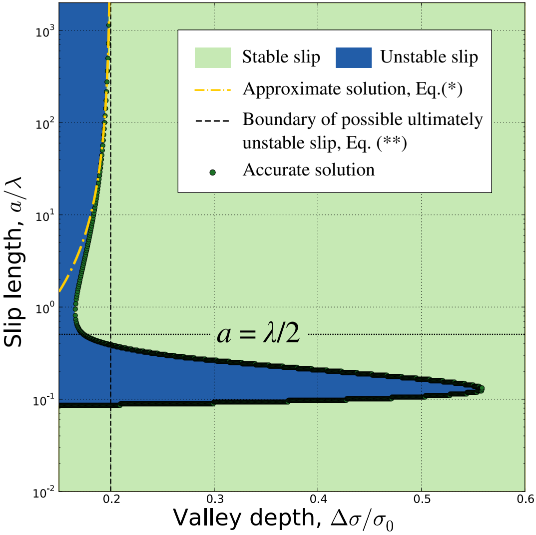
$$\frac{\Delta\sigma}{\sigma_0} \le \frac{f_0 - f_k}{f_k} \qquad (**)$$

- The corresponding slip length

$$a_u = \frac{\mu d_c(f_s - f_k)(1 + \Delta\sigma/\sigma_0)}{\pi\sigma_0(1-\nu)[f_0 - f_k(1 + \Delta\sigma/\sigma_0)]^2} \qquad (*).$$

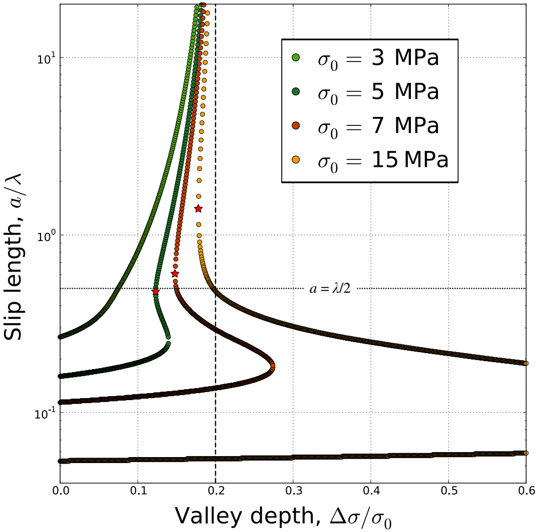
Effect of the pressure-valley depth $\Delta\sigma$

- Stability map for $\sigma_0 = 10$ MPa, $f_0 = 1.2 f_k$, $f_k = 0.6$, $f_s = 0.8$



Effect of the minimal pressure σ_0

- Stability map for $\sigma_0 = \{3,\ 5,\ 7,\ 15\}$ MPa, $f_0 = 1.2 f_k$, $f_k = 0.6$, $f_s = 0.8$



Perspective

- The slip length given by

$$a_u = \frac{\mu d_c(f_s - f_k)(1 + \Delta\sigma/\sigma_0)}{\pi\sigma_0(1-\nu)[f_0 - f_k(1 + \Delta\sigma/\sigma_0)]^2}$$

 represents only the upper limit for the ultimately unstable slip length

- For $\Delta\sigma/\sigma_0 \to (f_0 - f_k)/f_k$, slip length a_u diverges as

$$a_u \sim [f_0 - f_k(1 + \Delta\sigma/\sigma_0)]^{-2}$$

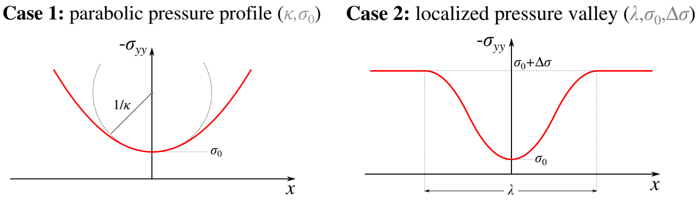
- Need a more accurate expression, which requires analysis of

$$\frac{\partial K_{II}}{\partial a} = 0$$

 where $K_{II}(a) = \left\langle \sqrt{\pi a}\tau_0 - J_1(\lambda/4, a) - J_2(\lambda/2, a) - J_3(a, a) \right\rangle$

Conclusions

- Some quantitative (approximate) results were obtained for two cases of non-uniform pressure distributions
  1. parabolic
  2. localized pressure valley



- Is unstable slip possible? *answered for parabola*
- When does the unstable slip arrest? *answered for parabola*
- What is the necessary condition for the slip to be ultimately unstable? *answered for valley*

Thank you for your attention!