The contact of elastic regular wavy surfaces revisited

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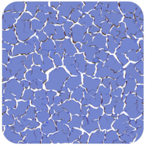
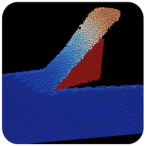

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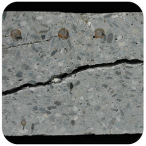
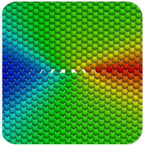
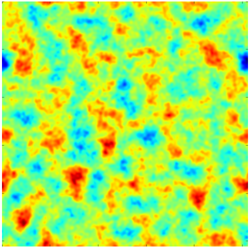
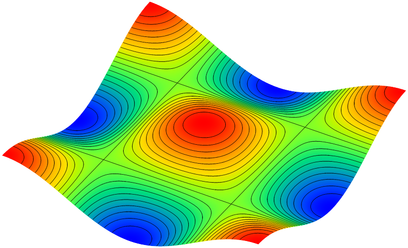




Motivation

Roughness

- Roughness topologies play important role in contact mechanics (friction, adhesion, etc...)
- Self-affinity is often used to describe roughness
- The simplest possible 3D roughness is bi-sinusoidal





Variational approach

$$min \quad f = \frac{1}{2}\int_S pu(p)dS + \int_S pg(p)dS$$

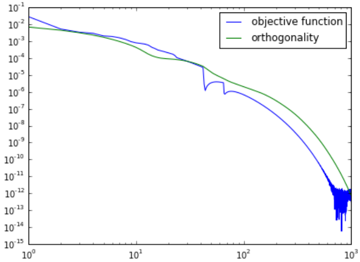
$$p \geq 0 \qquad \frac{1}{A_0}\int_S pdS = p_0$$

SQP approach

$$\nabla f = u + g$$

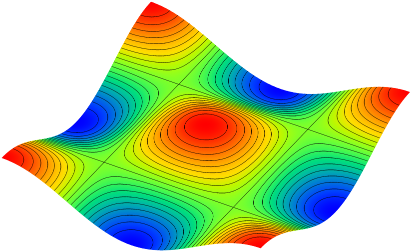
Enforce $\int_S pdS = A_0 p_0$ by dichotomy

Fourier space computation of influence functions
Convergence depends on #points



Problem settings

- Isotropic material $E^{\star}$
- small deformations
- Frictionless, non-adhesive contact
- Discretization 4096^2 points
- Periodic boundary conditions
- 200 load steps
- until full contact



$$z(x,y) = B cos(2\pi x/\lambda) cos(2\pi y/\lambda)$$

Johnson, Greewood, Higginson. Int.J.Mech.Sci. 27, 1985.
Krithivasan, Jackson. Tribol.Lett. 27, 2007

@ infinitesimal contact

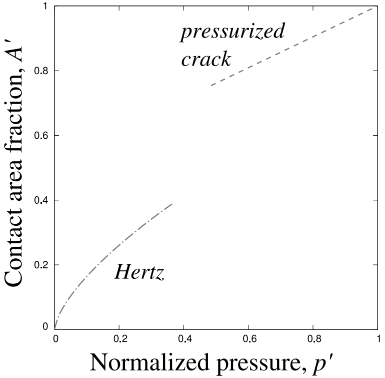
- Hertz theory
- Curvature is $R = 4\pi^2 B/\lambda^2$

$$A' = \pi \left(\frac{3p_0}{8\pi p^\star}\right)^{\frac{2}{3}}$$

Near full contact

- Pressurized crack assumption (Greenwood, IJSS, 56, 2015)

$$A' = 1 - \frac{3}{2\pi}\left(1 - p'\right)$$



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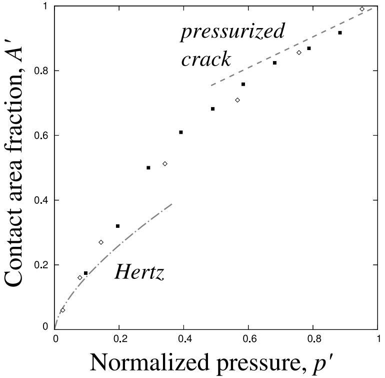

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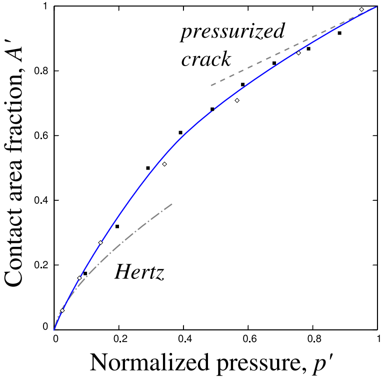

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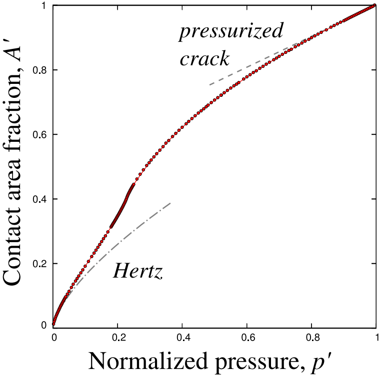

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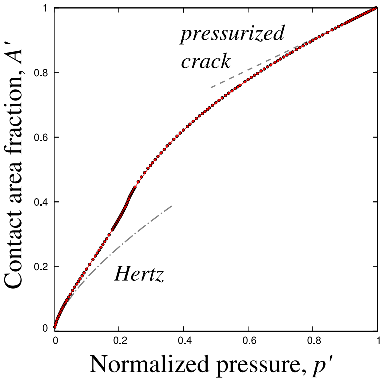
Convexity change

- Contact area variation has 2 inflexion points
- Related to two (unexpected) extrema of the mean pressure

Overlooked

- Numerical restrictions did not provide intermediate points
- Why is mean pressure droping?



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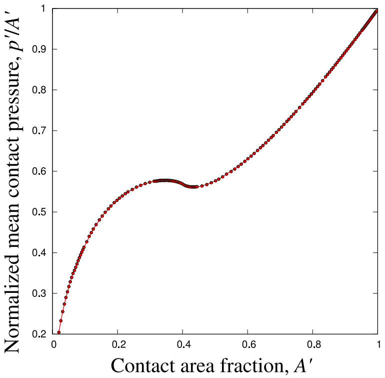
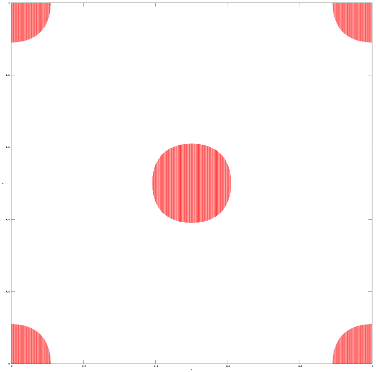


Shape of the contact zone

- Initially hertzian
- Becomes square-like : Loss of convexity
- Merging of contact zones
- Contact area grows more rapidly that pressure



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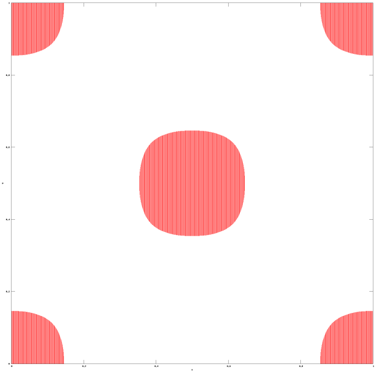

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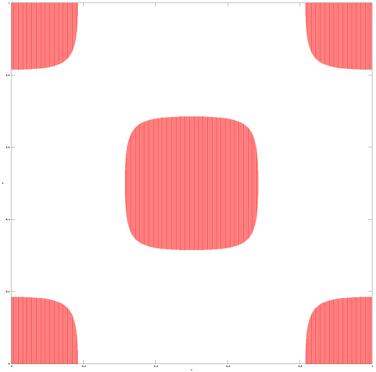

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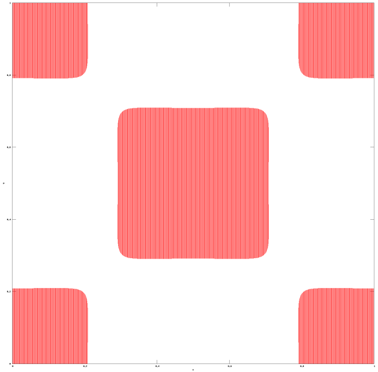

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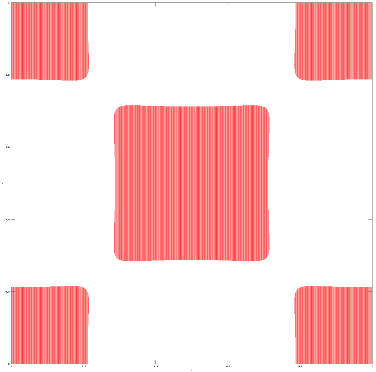

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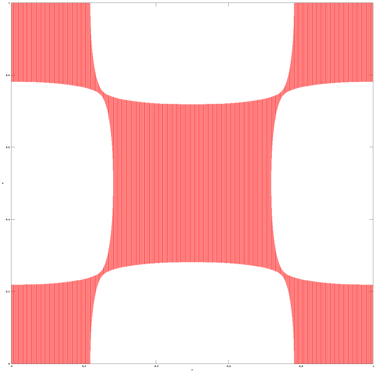

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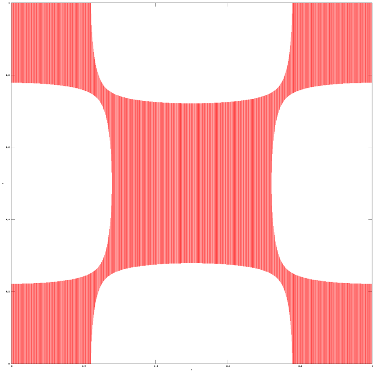

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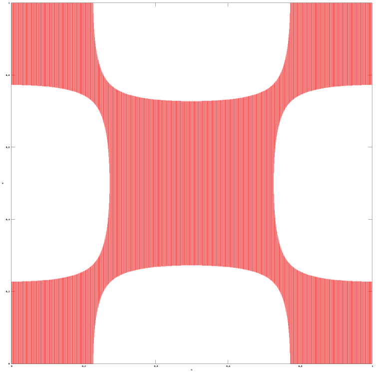

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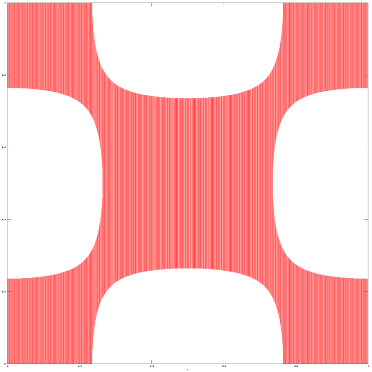

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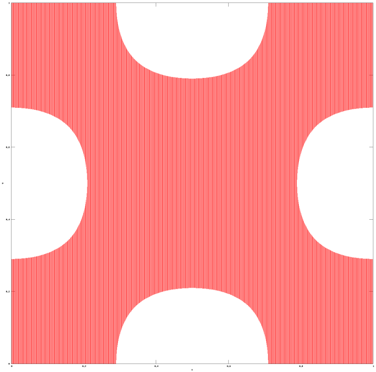

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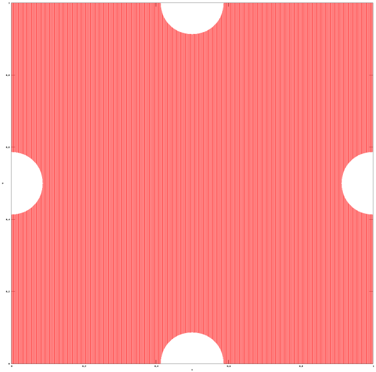

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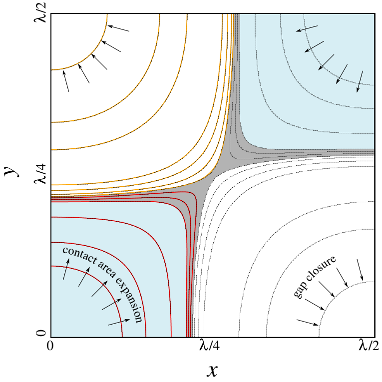

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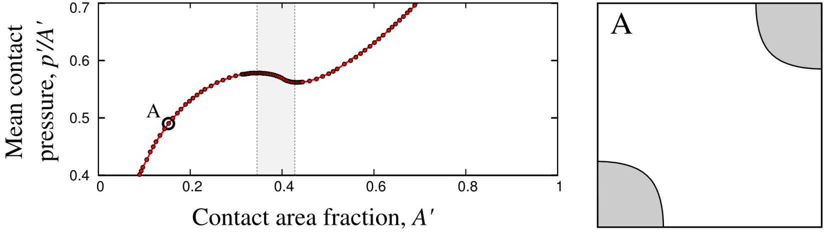

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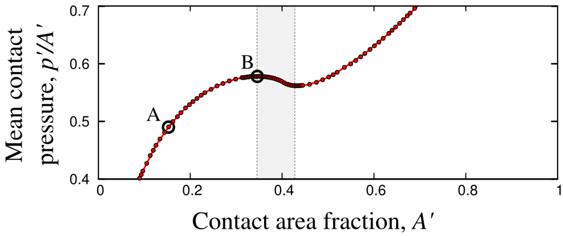

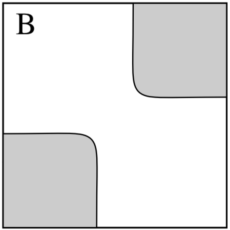

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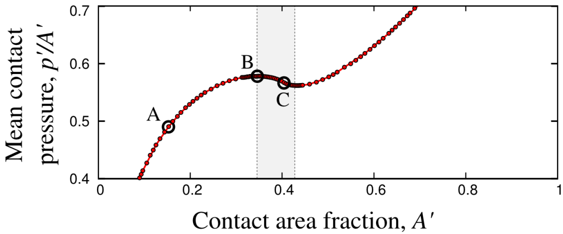

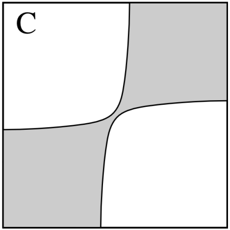

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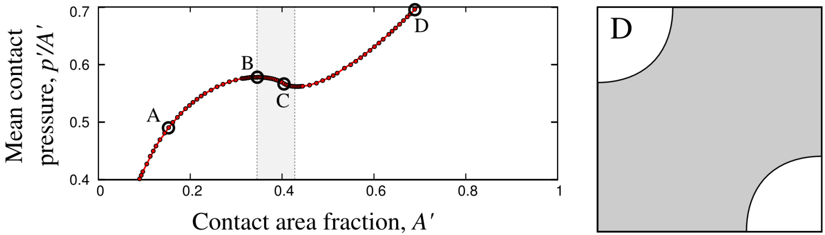
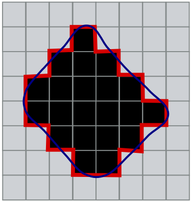


Perimeter evaluation

$S = \#contact\ transitions$



Compactness evaluation

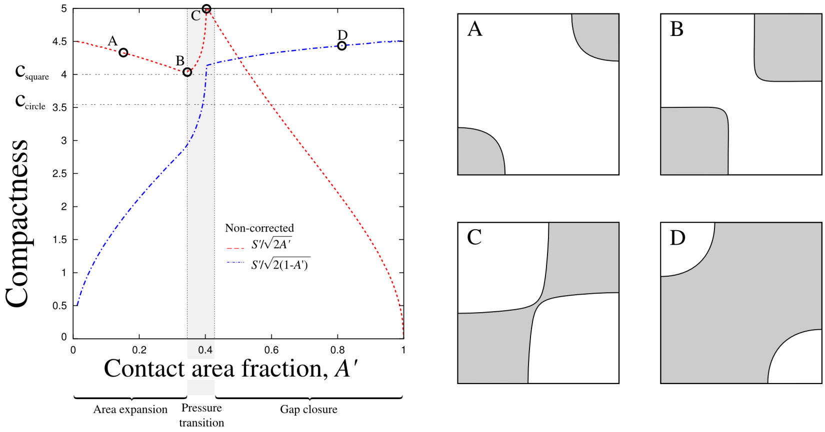
$\mathcal{C} = \frac{S}{\sqrt{A}}$

$\mathcal{C}_{circle} = 2\sqrt{\pi}$

$\mathcal{C}_{square} = 4$

Contact perimeter

Compactness of contact area



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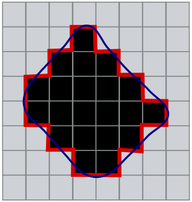

Compactness evaluation

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Properties of the perimeter

- Wrong if curved
- Exact if square

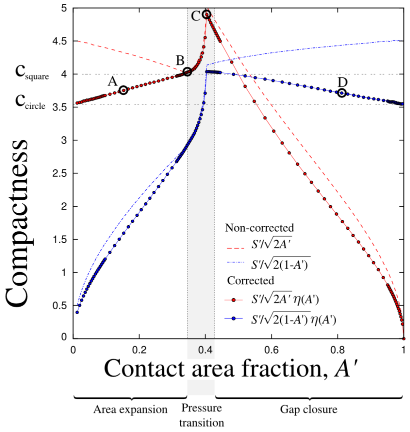
Correction of the perimeter

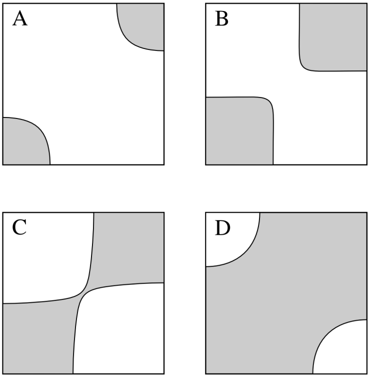
$$S = \frac{S^d}{\eta(A')n}$$

- S^d the "discrete" perimeter
- $\eta(A')$ interpolates from circle to square compactnesses

Contact perimeter

Compactness of contact area

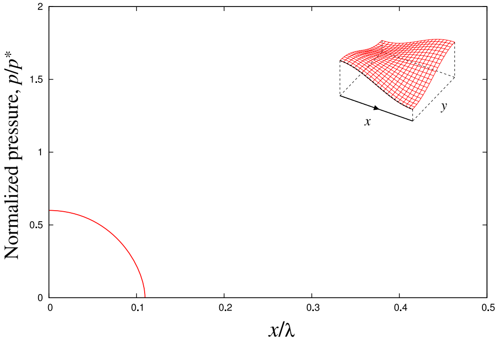




Contact pressure

Westergaard

$$p(x,a) = 2p_0 \frac{cos(\frac{\pi x}{\lambda})}{sin^2(\frac{\pi a}{\lambda})} \sqrt{sin^2(\frac{\pi a}{\lambda}) - sin^2(\frac{\pi x}{\lambda})}$$



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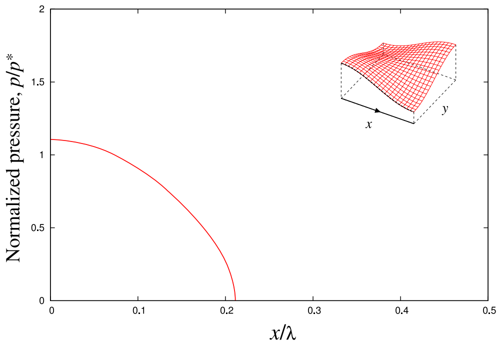

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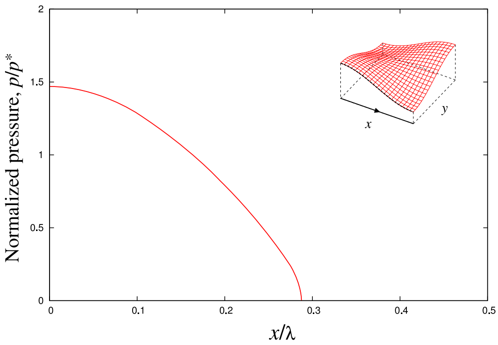

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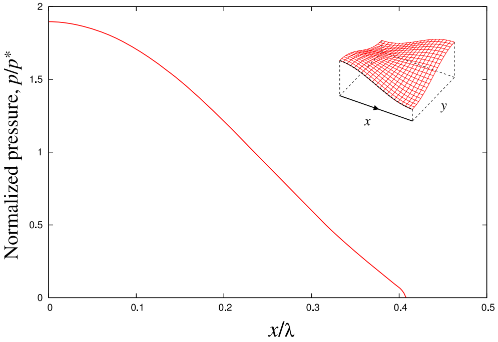

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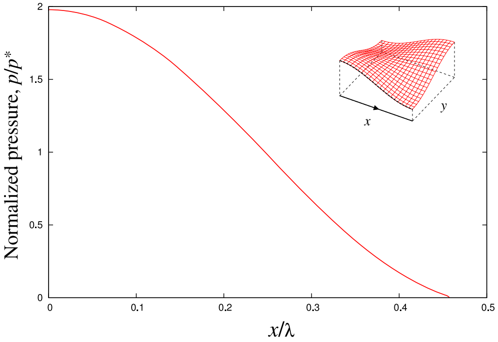
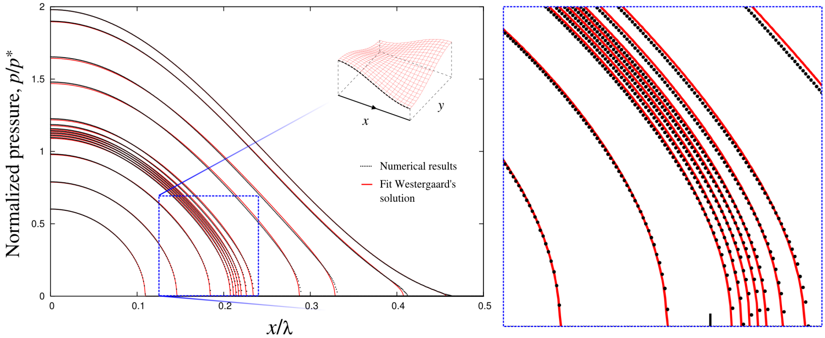


Contact pressure

Simulation pressure profiles

- Fits the assymptotic values
- Junction cancels the pressure profile slope



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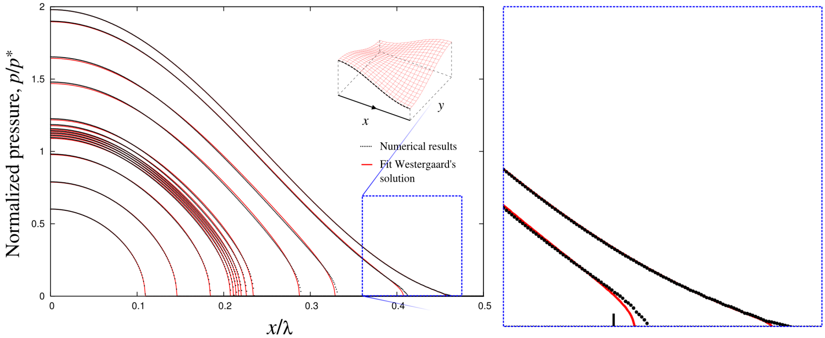

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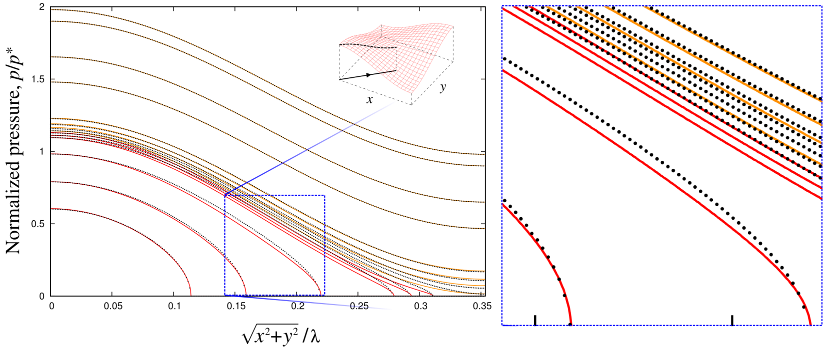

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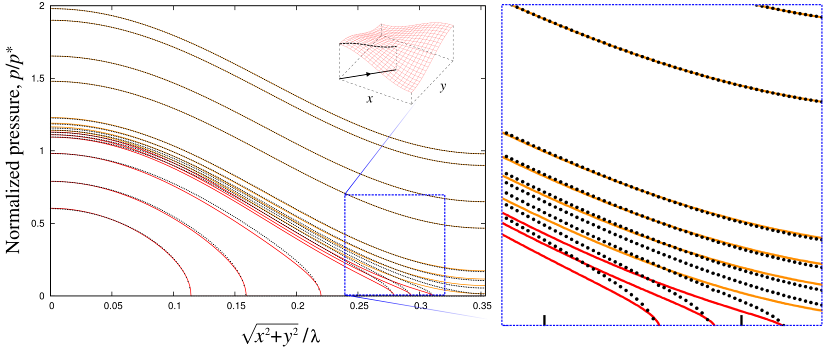


Probability density of contact pressure

PDF of pressures

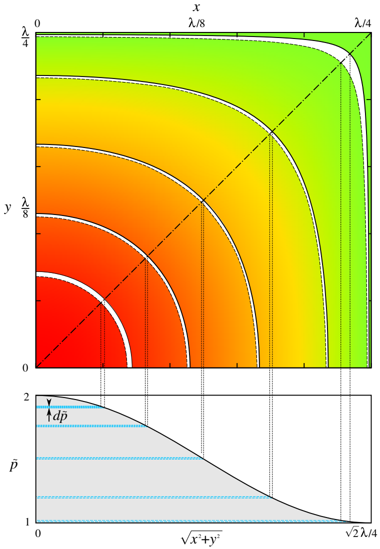
$$P(\tilde{p}) = \frac{1}{A_0} \int_{A_0} \delta(\tilde{p} - p(x,y))dxdy$$

- Property

$$\int_{\tilde{p}} P(\tilde{p})d\tilde{p} = A_0/A_0 = 1$$

Numerical measure

- Decomposition in bins
- Intractable PDF function at the limit to zero



Probability density of contact pressure

Hertz analytic solution at small contact

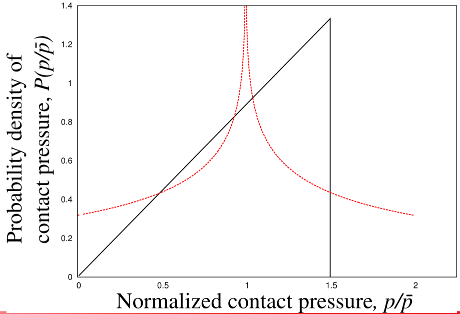
Hertz:

$$P(\tilde{p}, p_0) = \frac{8}{9}\tilde{p}$$

Full contact:

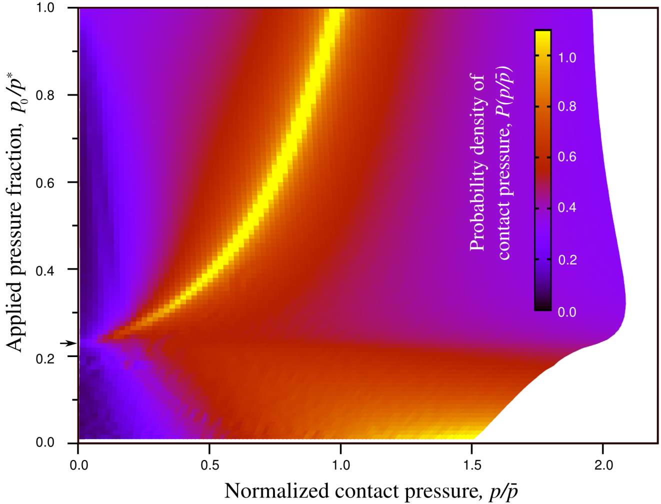
$$P(\tilde{p}) = \frac{4}{\pi}\frac{F(arccos(\tilde{p}-1), 1/\sqrt{2\tilde{p}-\tilde{p}^2})}{\sqrt{2\tilde{p}-\tilde{p}^2}}$$

$$with \quad F(l,k) = \int_0^l 1/\sqrt{1-k^2 sin(x)}dx$$



Probability density of contact pressure

Evolution with load



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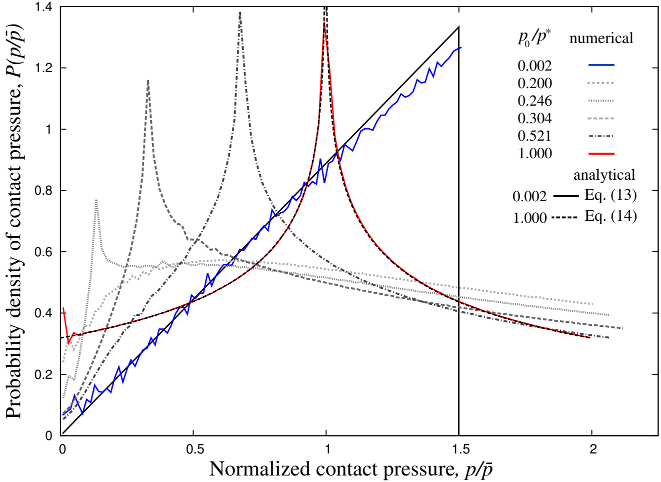


Implication for Persson's model

Persson's model (elastic case)

- Manipulate the Probability density function $P(p, \zeta)$
- as a Function of the applied pressure p and magnification ζ
- Under full contact assumptions we obtain

$$\frac{\partial P(p,\zeta)}{\partial V} = \frac{1}{2}\frac{\partial^2 P(p,\zeta)}{\partial^2 p}$$

- where V is the variance of the pressure distribution.
- V is approximated by Persson as the variance achieved at full contact (elastic correlation to the heights profile):

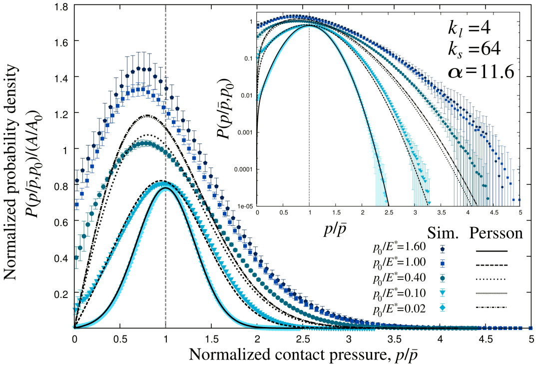
$$V = \frac{1}{2}E^\star m_2(\zeta) = \frac{\pi E^\star}{2}\int_{k_1}^{\zeta k_1} k^3 \Phi^p(k) dk$$

Greenwood and Manners. Some observations on Persson's diffusion theory of elastic contact. Wear 261, 2006

Persson's assumptions

- In the derivation of the diffusion equation: full contact is assumed
- Need for a boundary condition to precise solution:

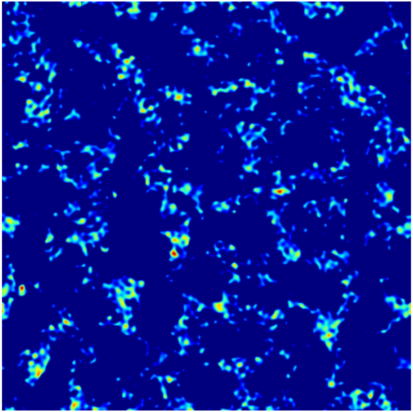
$$P(p = 0, \zeta) = 0$$



How is the wavy surface result supposed to impact Persson's assumption ?

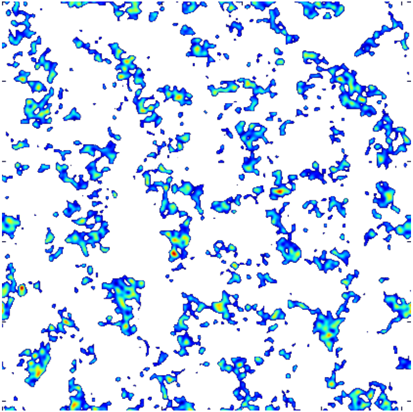
Splitting the contact spots

$$P(\tilde{p}) = \frac{1}{A_0} \int_{A_0} \delta(\tilde{p} - p(x,y)) dxdy$$



Splitting the contact spots

$$P(\tilde{p}) = \frac{1}{A_0}\int_A \delta(\tilde{p} - p(x,y))dxdy + \frac{A_0 - A}{A_0}\delta(p)$$

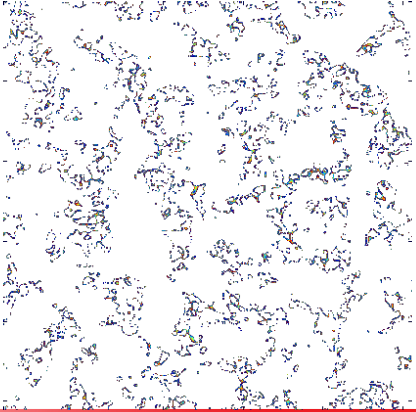


Implication for Persson's model

Persson's assumptions

We are interested in the region $0 < \tilde{p} < \epsilon$:

$$P(\tilde{p}) = \frac{1}{A_0} \sum_{i}^{N(\epsilon)} \int_{A_i(\epsilon)} \delta(\tilde{p} - p(x,y))dxdy$$



We want to investigate the limit when $\epsilon \to 0$:

$$P(\tilde{p}) \to \frac{1}{A_0} \sum_{i}^{N(0^+)} \Gamma(p_0')$$

where $\Gamma(p_0')$ is the pdf of patches of contact

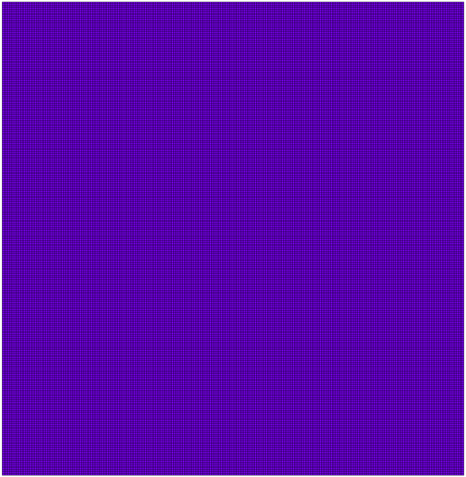
One should estimate the spatial density of asperity merging sites $D(p'_0)$ for which $\Gamma(p'_0) \neq 0$:

$$P(\tilde{p} = 0+) = \frac{1}{A_0} \sum_{i}^{D(p'_0)A_0} \Gamma(p'_0) \simeq D(p'_0)\Gamma(p'_0)$$

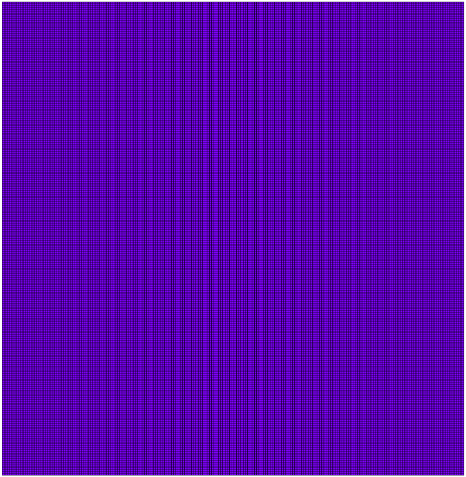
Missing ingredients

- $\Gamma(p'_0)$ the average PDF of pressure for merging asperities
- $D(p'_0)$ the spatial density of asperities merging at applied pressure p'_0

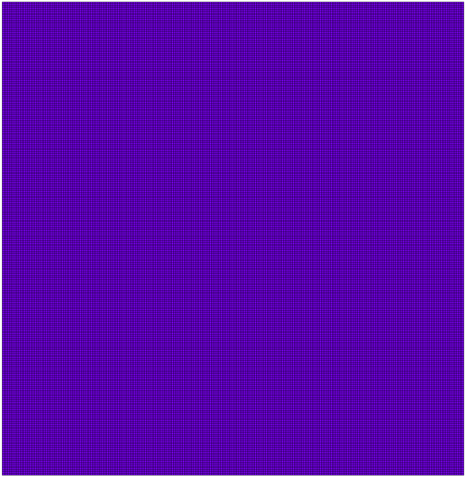
Simulation Zoom (1/64)



Simulation Zoom (1/64)



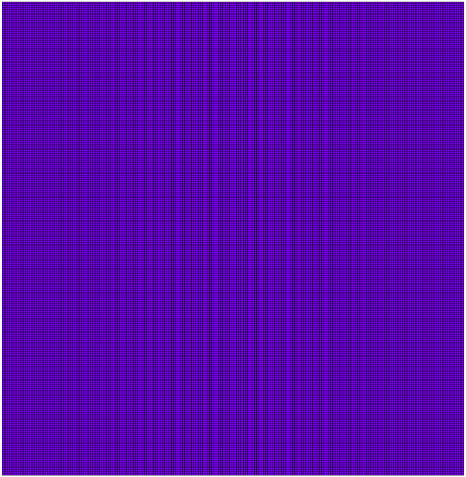
Simulation Zoom (1/64)



Implication for Persson's model

Persson's assumptions

Simulation Zoom (1/64)



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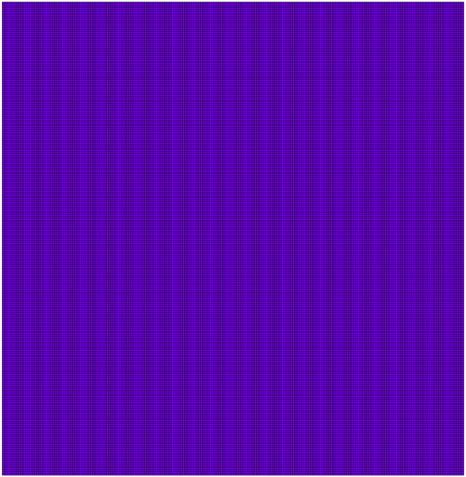

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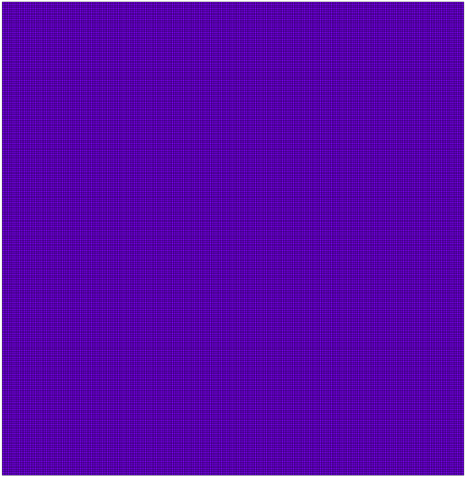

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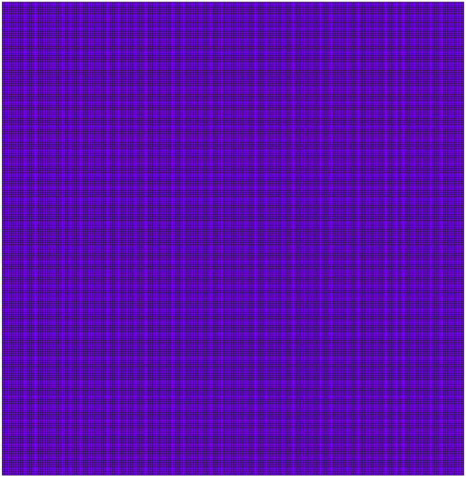

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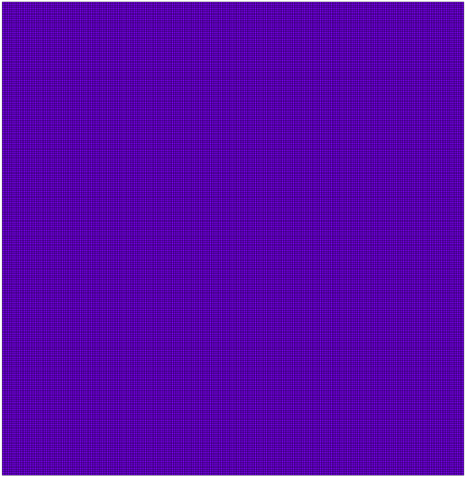

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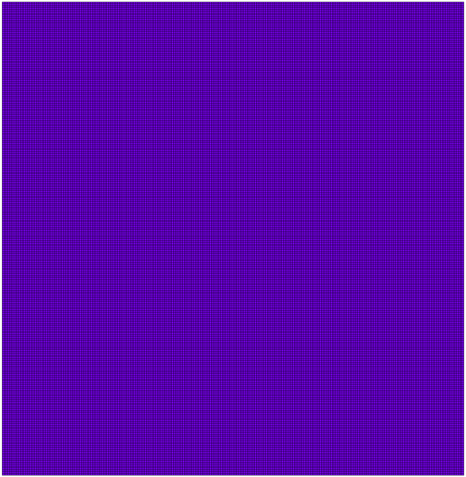

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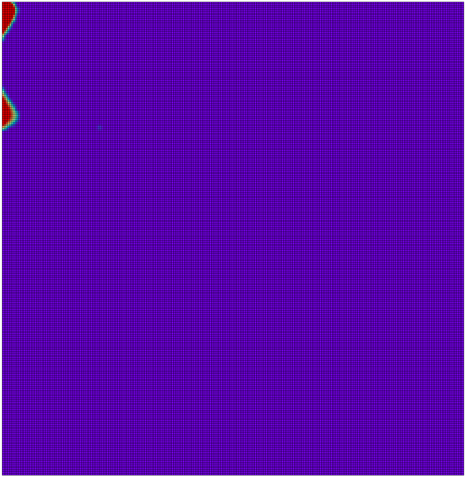
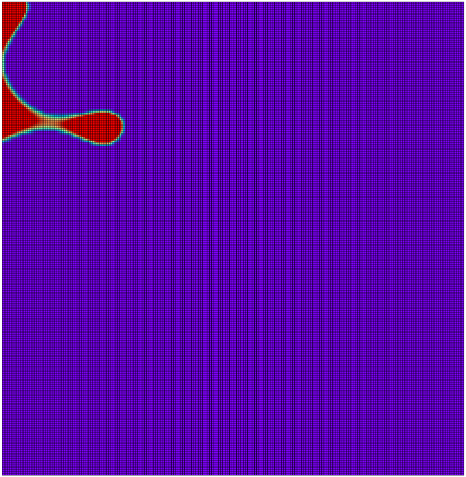


Implication for Persson's model

Persson's assumptions

Simulation Zoom (1/64)



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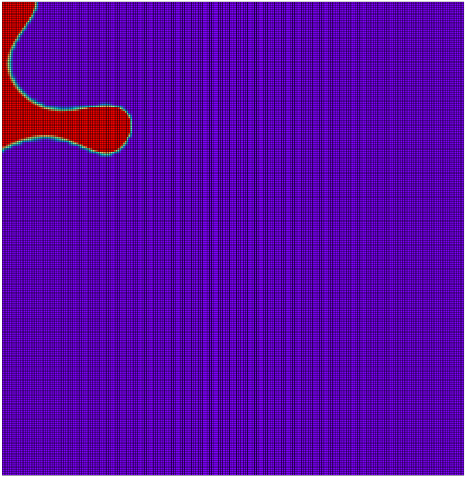

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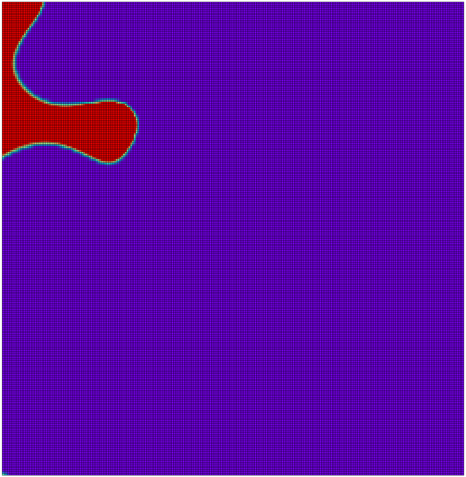

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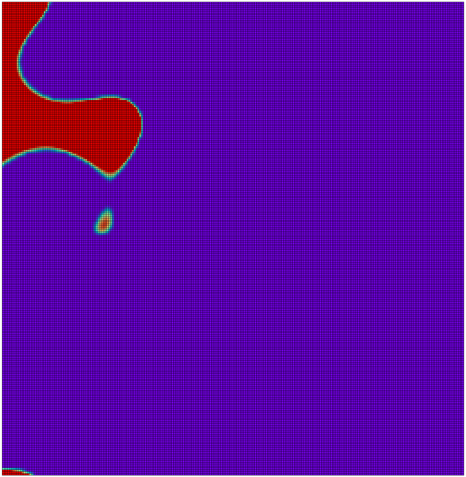

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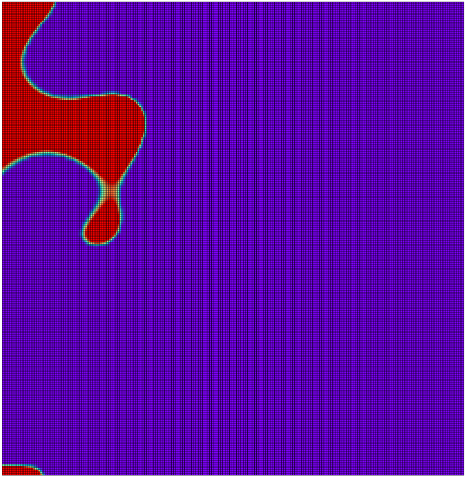

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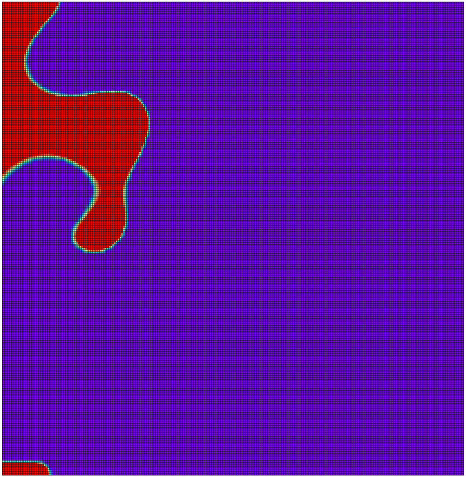

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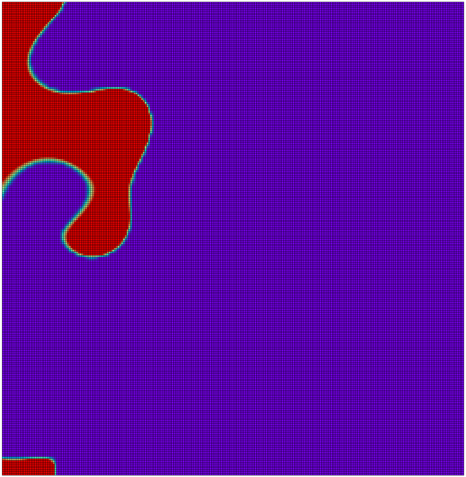

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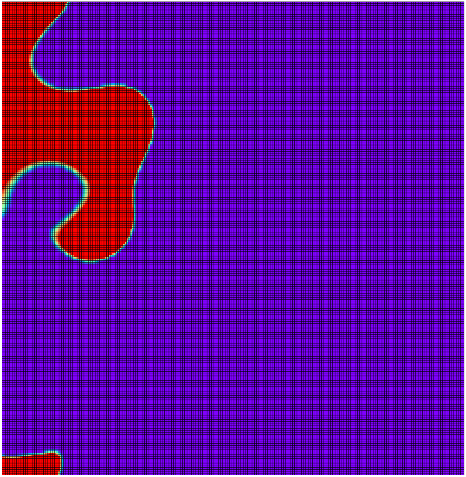

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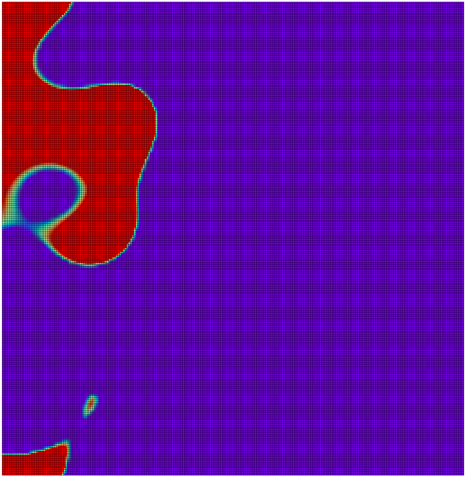

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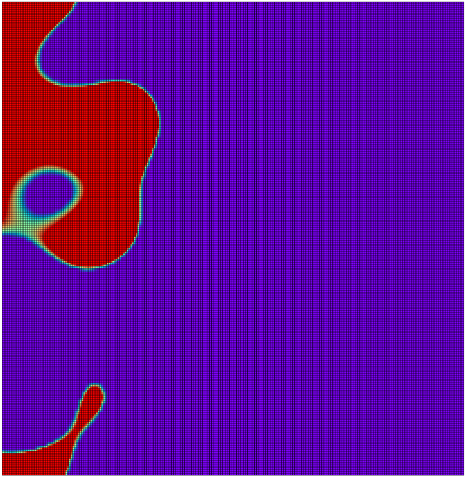

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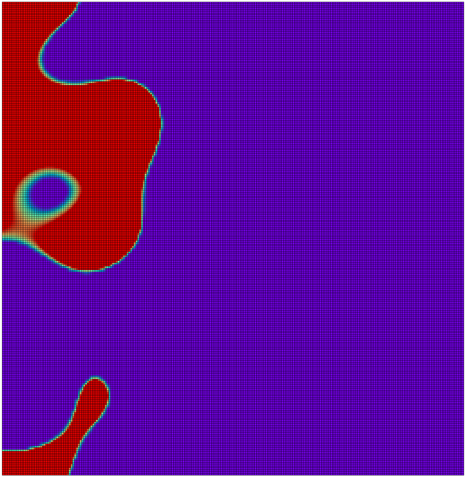

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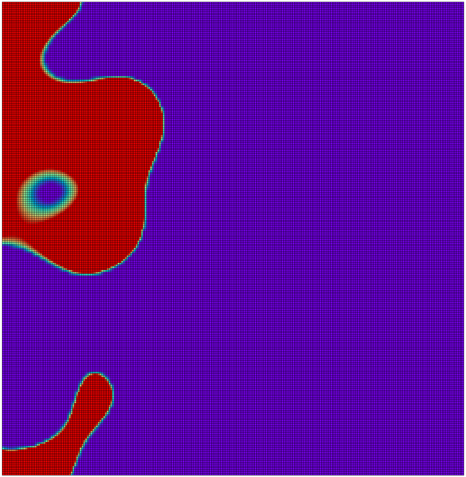

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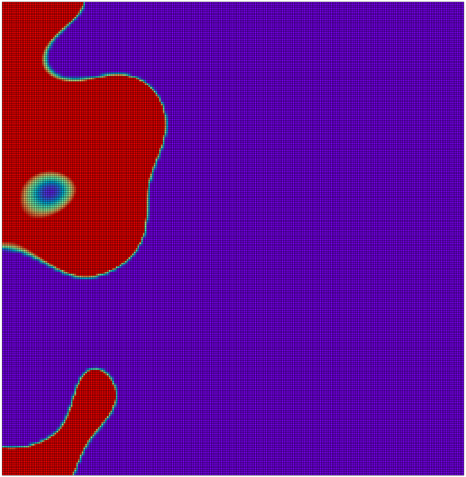

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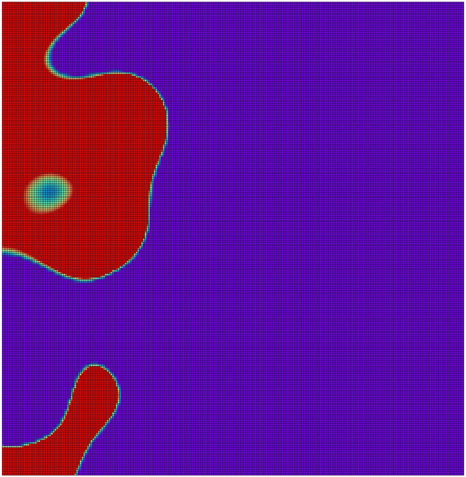

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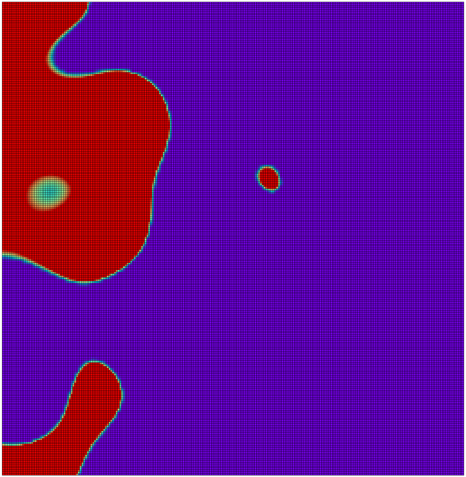

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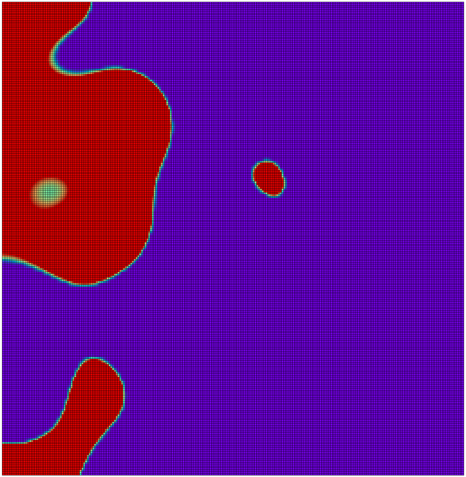

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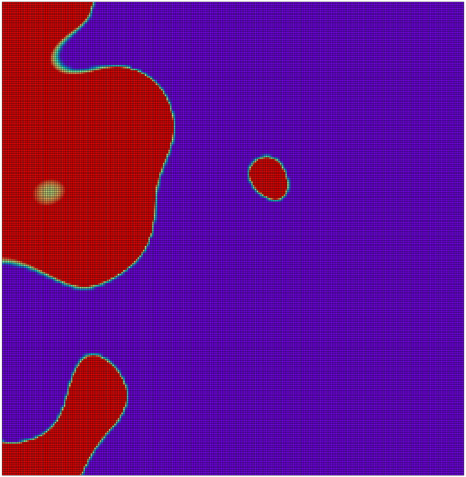

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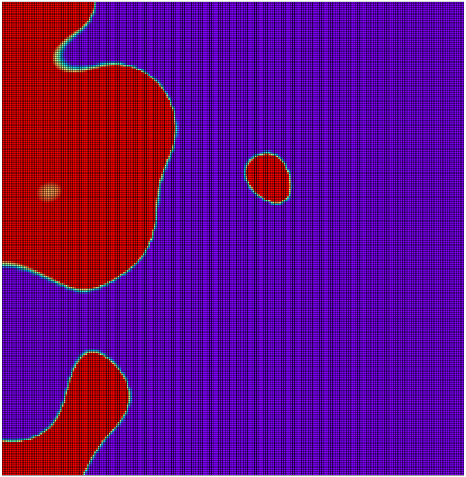

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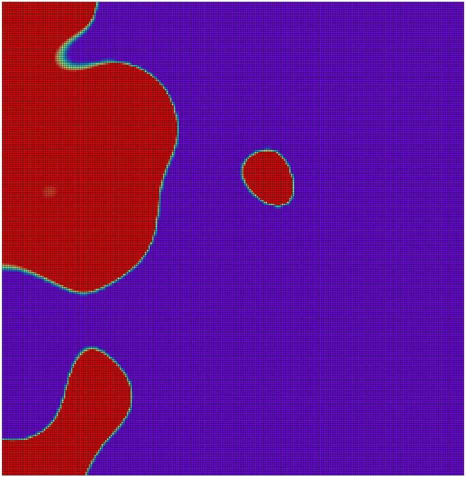

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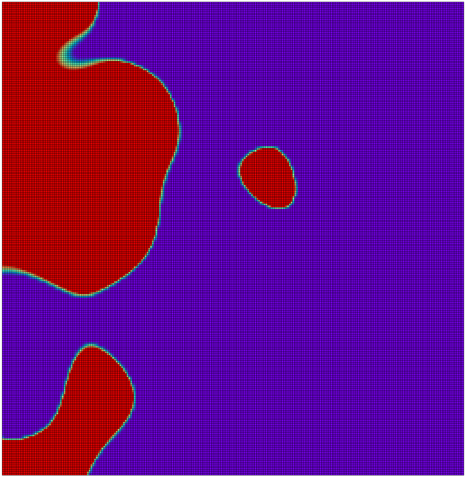

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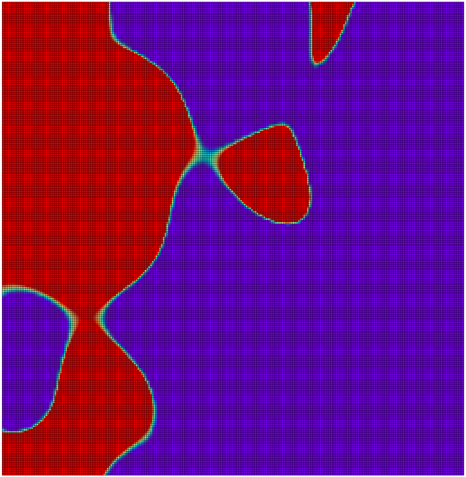

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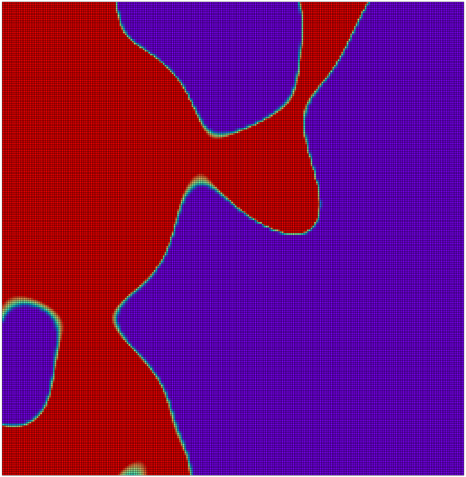

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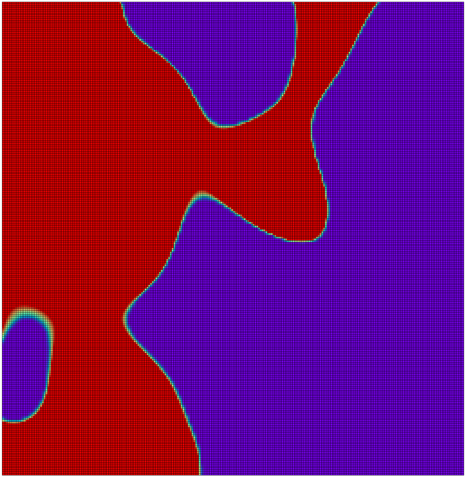

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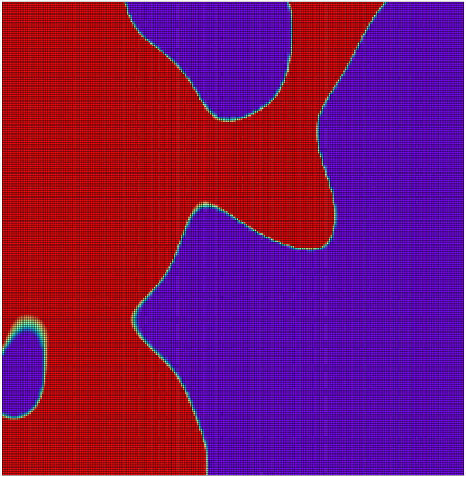

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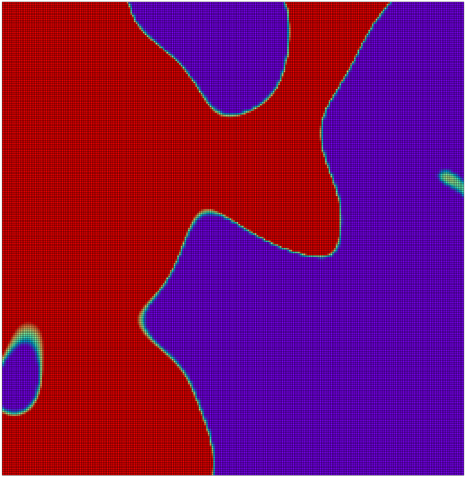

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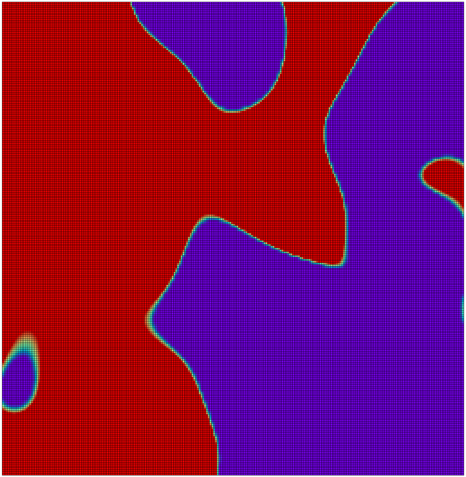

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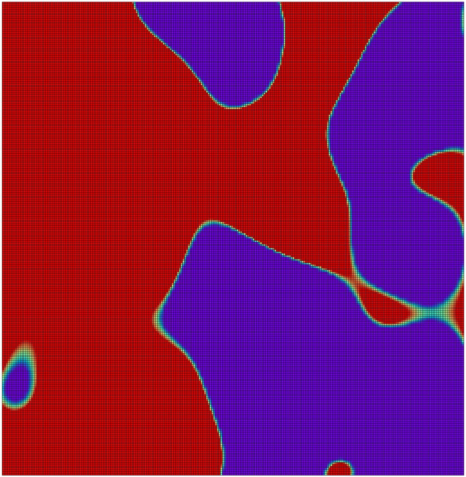

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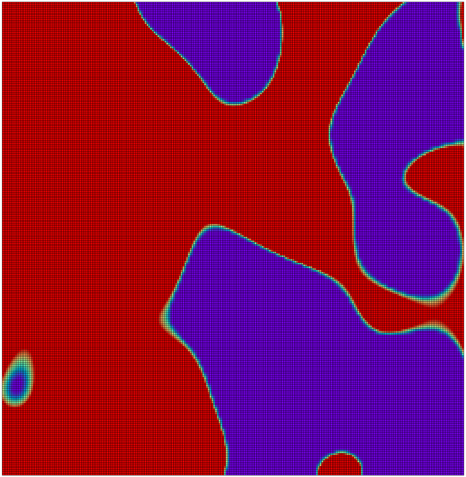

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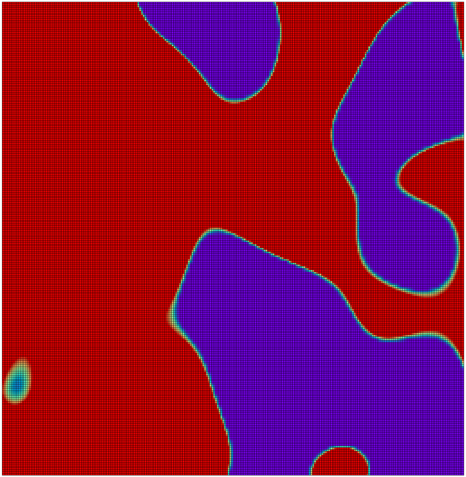

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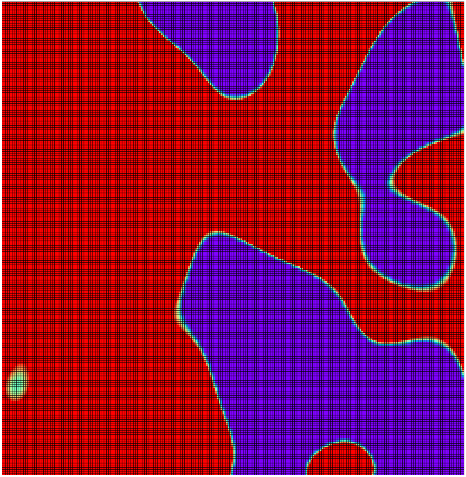

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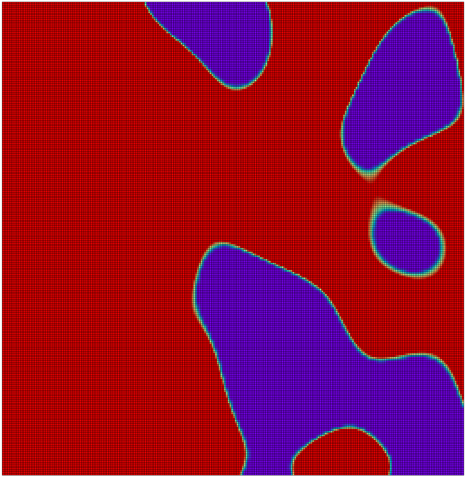

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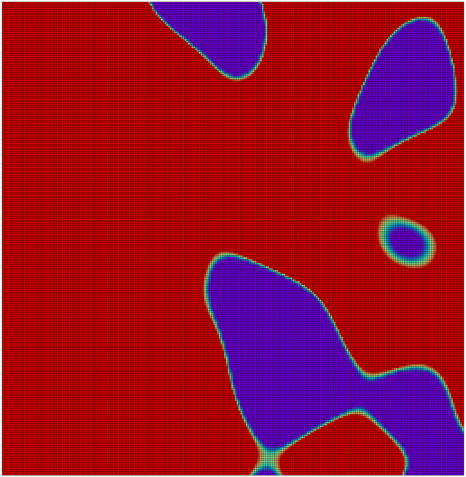

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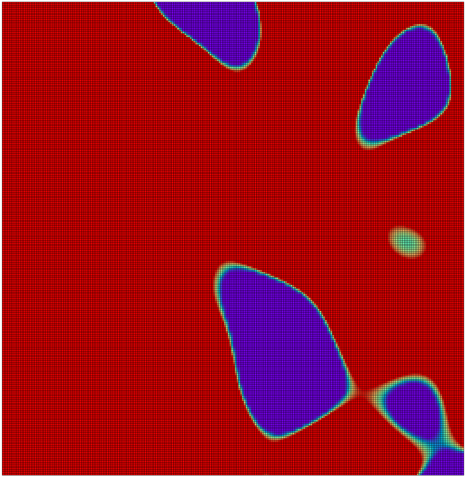

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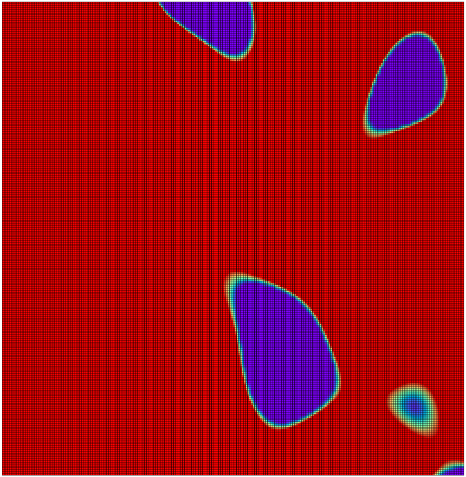

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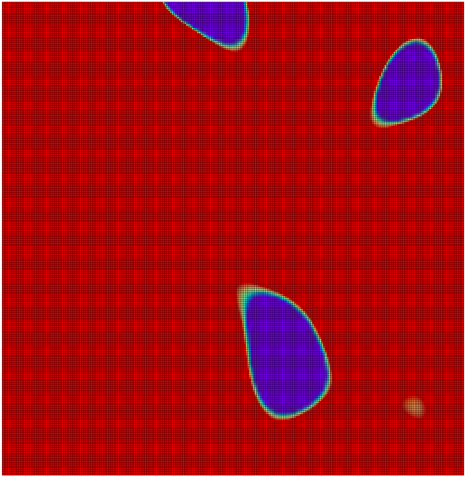

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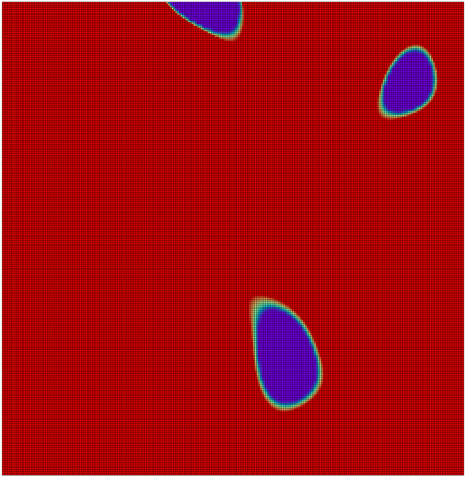

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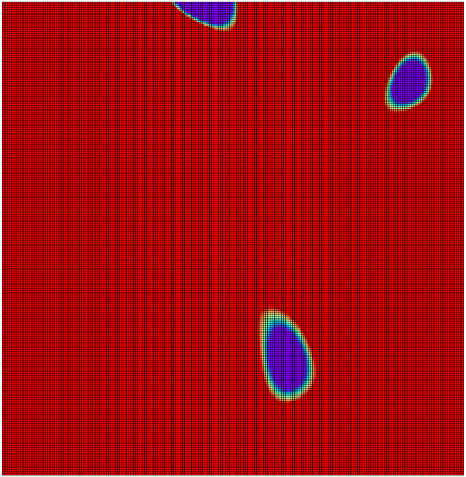

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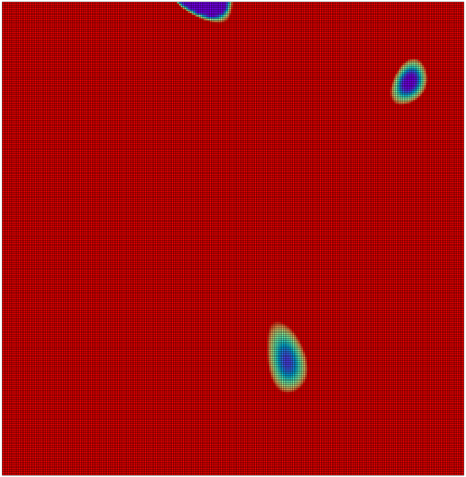

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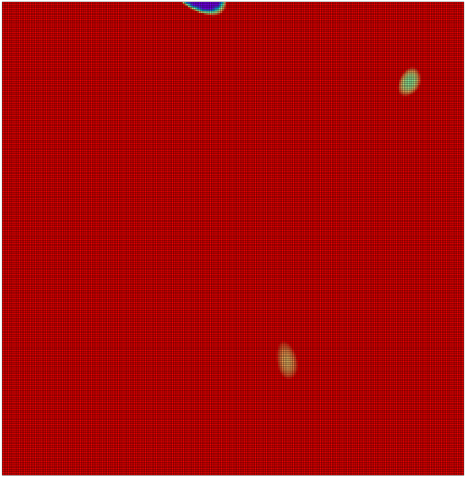

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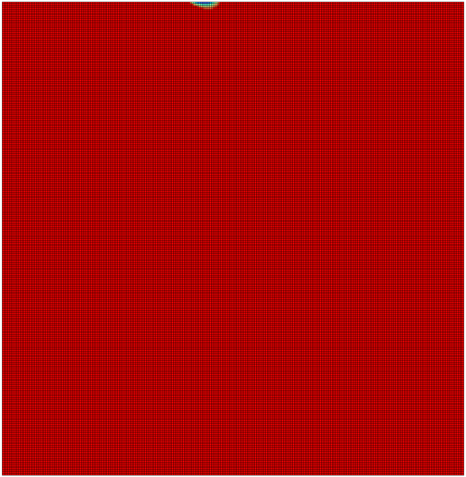
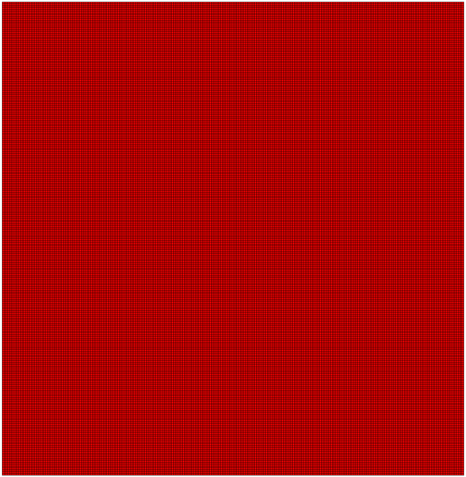


Implication for Persson's model

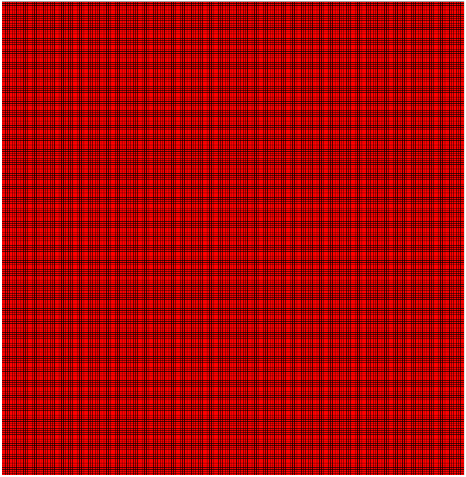
Simulation Zoom (1/64)



Implication for Persson's model

Persson's assumptions

Simulation Zoom (1/64)



Implication for Persson's model

Persson's assumptions

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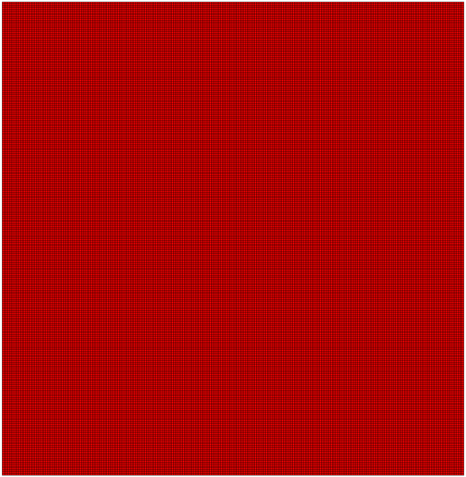

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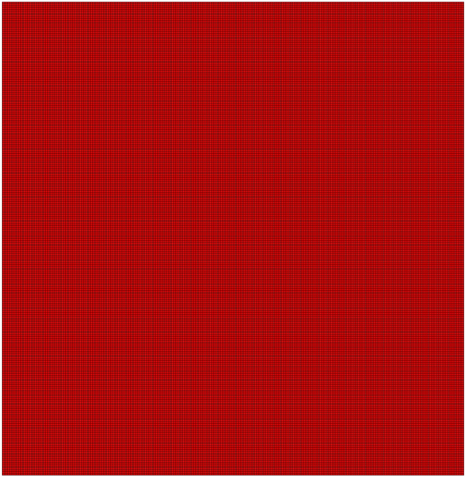

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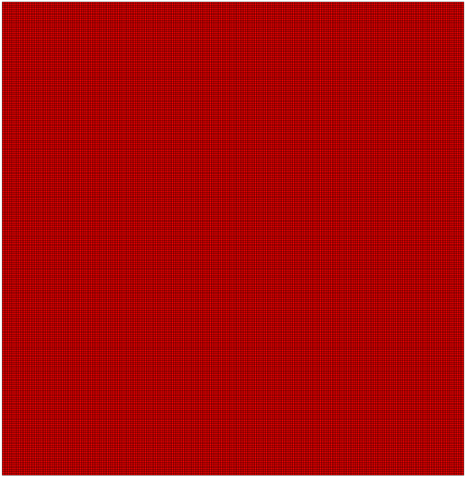

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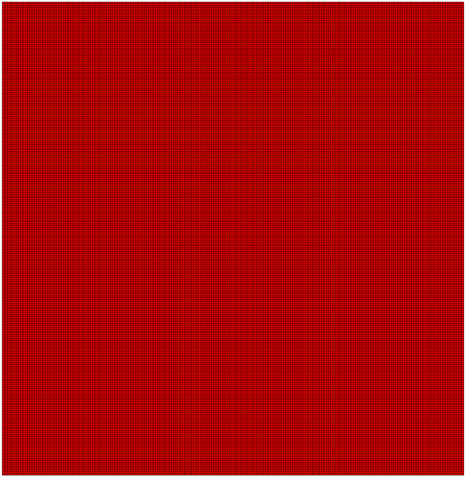

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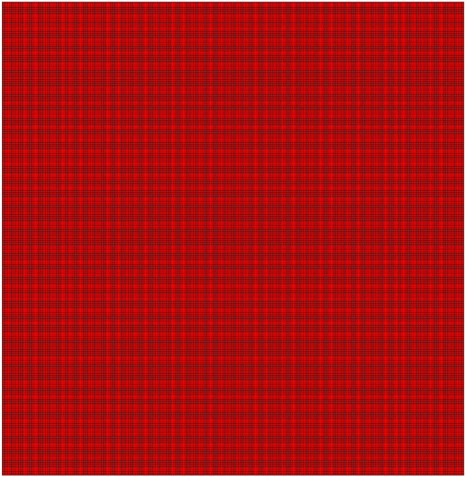

Implication for Persson's model

Persson's assumptions

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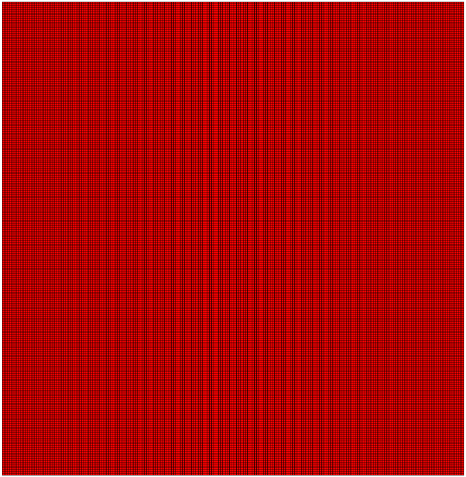

Implication for Persson's model

Persson's assumptions

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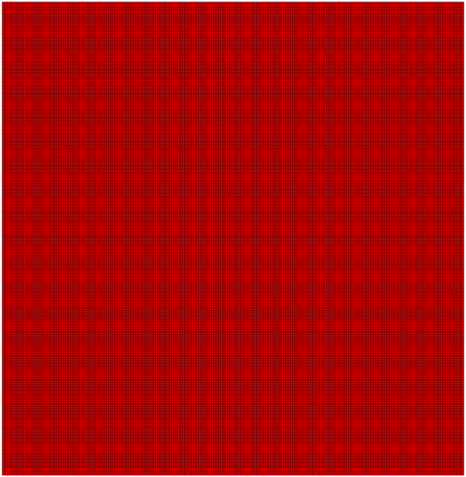

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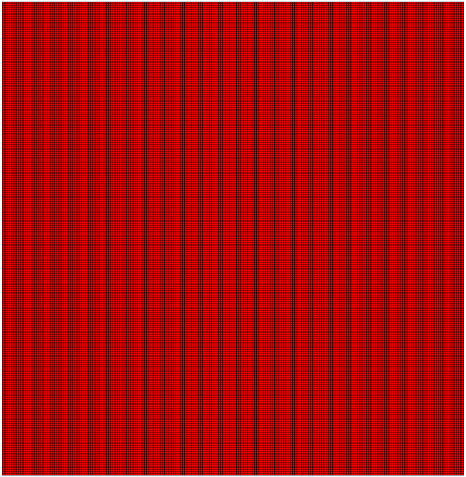

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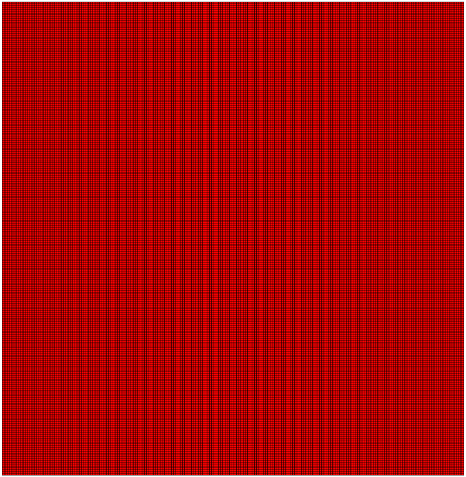

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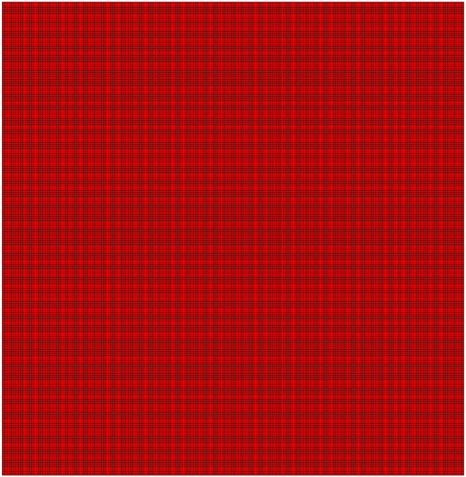
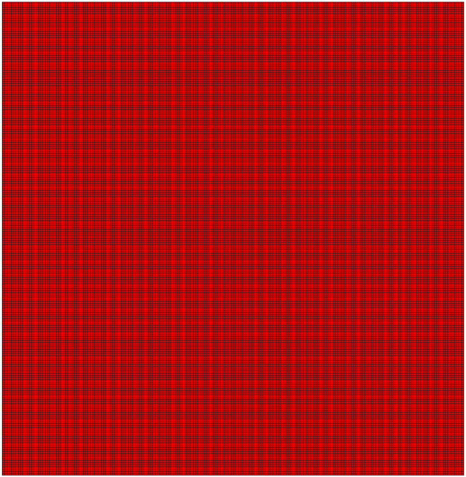
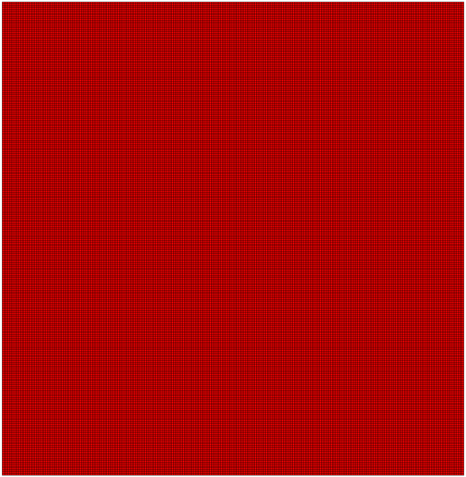


Simulation Zoom (1/64)



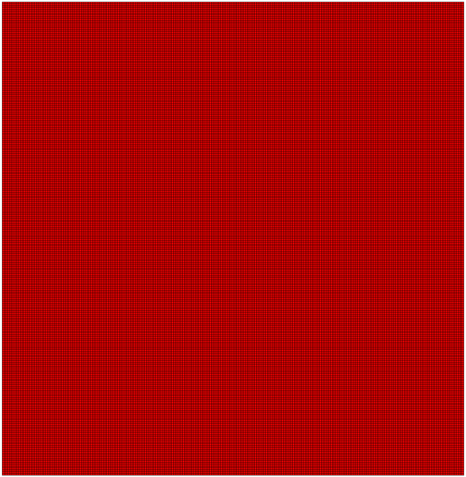
Simulation Zoom (1/64)



Implication for Persson's model

Persson's assumptions

Simulation Zoom (1/64)



Conclusion

Overlooked particularity of bi-sinusoidal surface

- Area of contact: change of convexity
- Mean pressure: unexpected drop
- Perimeter: correction of numerical results is needed
- At full contact: $P(0^+) \neq 0$

V.A. Yastrebov, G. Anciaux, J.F. Molinari, The contact of elastic regular wavy surfaces revisited. Tribol.Lett. 56, 2014

Future work

- Is it generalizable that $P(0^+) \neq 0$?