Mechanical contact between rough elastic-plastic solids: *scale effect in asperity deformation*

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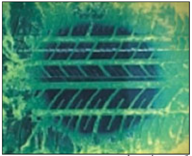
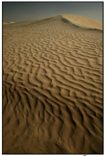
Society of Engineering Science 52nd Annual Technical Meeting
Texas A&M, College Station, USA
October 26–28, 2015

#Introduction

Contact / friction applications

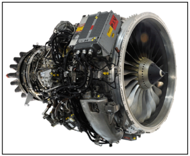

tire/road


granular


bearings


electric

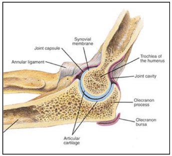

railroad


assembled pieces

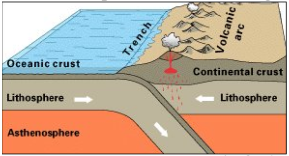

drilling


touch interfaces

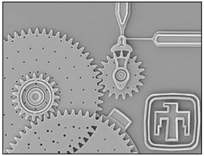

metal forming



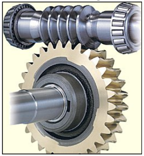
human joints



tectonic faults


MEMS


brakes


gears

Roughness

- **Natural and industrial surfaces are *rough*:**
  - processing
  - polishing
  - coating
  - microstructure
  - surface energy
  - deformation
  - aging
  - environment

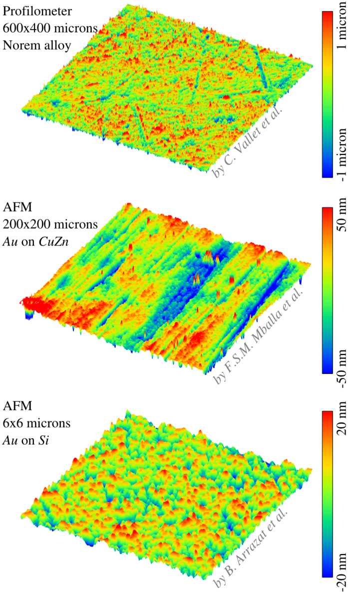


Fig. Examples of rough surfaces

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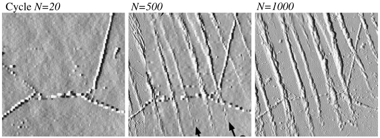

Fig. Persistent slip marks [1]

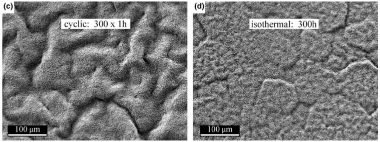

Fig. Rumpling (thermal cycling induced roughness in air)[2]

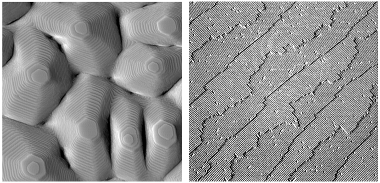

Fig. Epitaxial surface growth [3,4]

[1] J.Polák, J. Man & K. Orbtlík, Int J Fatigue 25 (2003)
[2] V.K. Tolpygo, D.R. Clarke, Acta Mat 52 (2004)
[3] M. Einax, W. Dieterich, P. Maass, Rev Mod Phys 85 (2013)
[4] J.R. Arthur, Surf Sci 500 (2002)

Roughness

- **Natural and industrial surfaces are *rough*:**
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- **Roughness affects:**
  - stress-strain state
  - friction
  - wear
  - adhesion
  - fluid flow
  - sealing
  - energy transfer

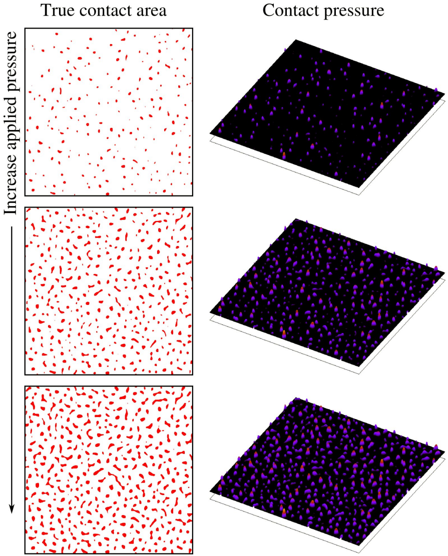

Fig. True contact area and stress fluctuations

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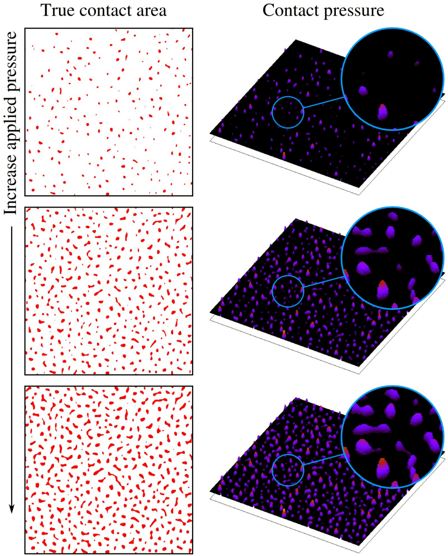

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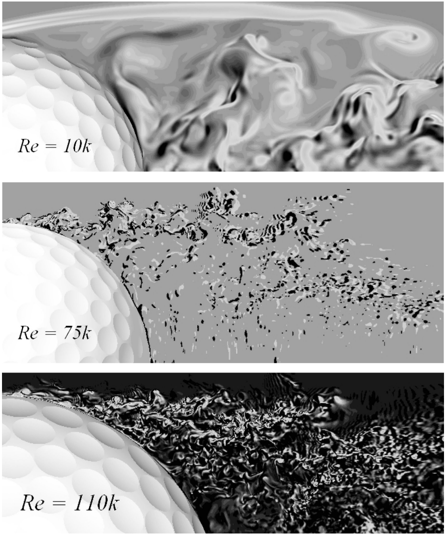

Fig. Numerical simulation of airflow around a (dimpled) golf ball [5]

[5] C.E. Smith, PhD thesis (2011)

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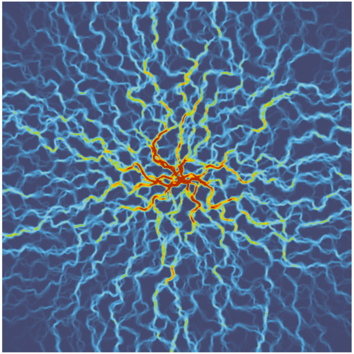

Fig. Fluid passage through free volume between rough surfaces

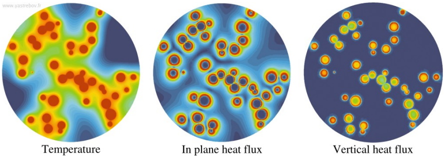

Fig. Heat transfer between rough surfaces (asperity-based model)

#Roughness

Roughness. Characteristics.

- Fractal (self-affine) roughness
- Power spectral density (PSD)

$$\Phi(k) \sim k^{-2(H+1)}$$

  k is a wavenumber,
  H is the Hurst exponent.
- **Gaussian**/non-Gaussian height distribution $P(h)$
- **Isotropic**/anisotropic surfaces

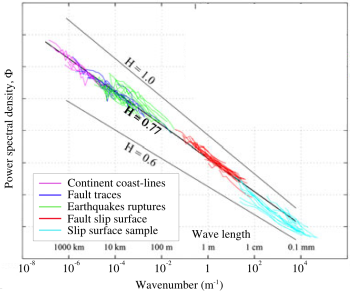

Fig. Power spectral density, geological scales

Adapted from
[4] Renard, Candela, Bouchaud, Geophys. Res. Lett. 40 (2013)

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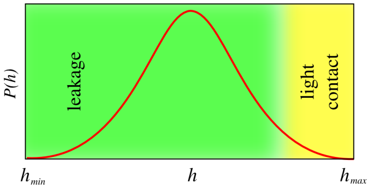

Fig. Height distribution $P(h)$

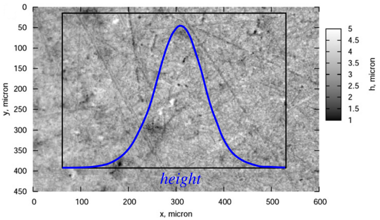

Fig. Height distribution of a polished metal surface

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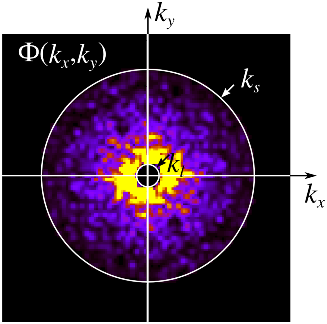

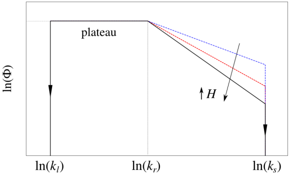

Fig. 3D and radial power spectral densities

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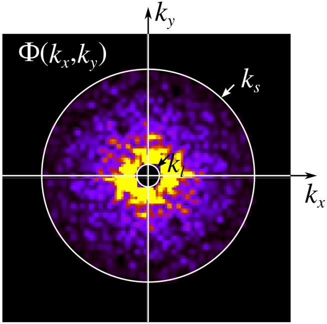
- **Gaussian**/non-Gaussian height distribution $P(h)$
- **Isotropic**/anisotropic surfaces
- **Characteristics:**
  - $\sqrt{\langle h^2 \rangle}$ - *std heights*
  - $\sqrt{\langle |\nabla h|^2 \rangle}$ - *std slope (surface gradient)*
  - $\alpha = m_{00} m_{40} / m_{20}^2$ - *breadth of the spectrum (Nayak's parameter*[B]*),*

  spectral moments $m_{pq} = \iint\limits_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y)\, dk_x dk_y$
- **Random process theory**



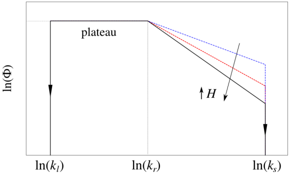


Fig. 3D and radial power spectral densities

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[C] Greenwood, Wear 261: 191 (2006)
[D] Borri, Paggi, J. Phys. D Appl Phys 48:045301 (2015)

Roughness. Characteristics.

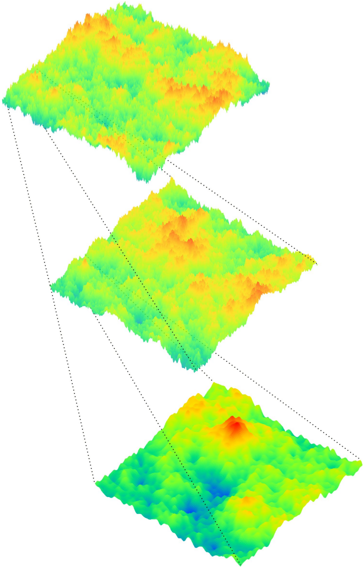


Fig. Example of a rough surface for $H = 0.3$

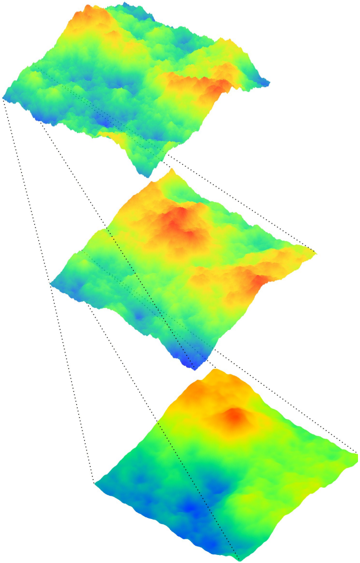


Fig. Example of a rough surface for $H = 0.8$

Recall: the Hurst exponent H and the fractal dimension D in 2D space are interconnected via $D = 3 - H$

[1] Hu, Tonder. *Int. J. Mach. Tool Manuf.* 32 (1992)

Roughness enhancement

- Data interpolation (Shanon, bi-cubic Bézier surfaces)

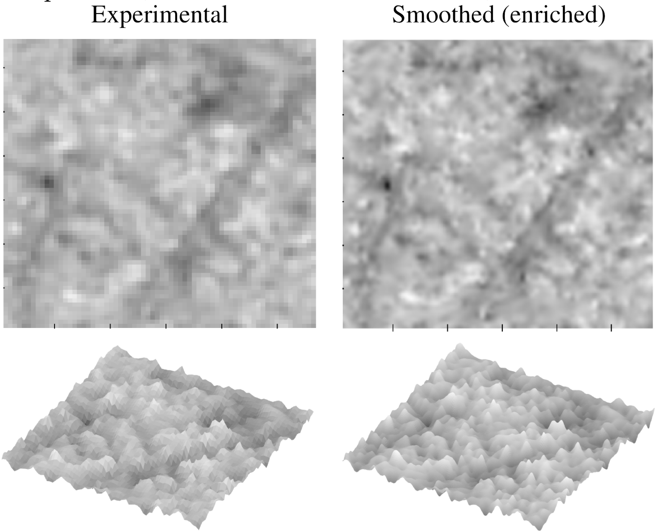


Fig. Bi-cubic Bézier interpolation of an experimental rough surface

[1]Hyun, Robbins. *Tribol. Int.* (2007)
[2] Yastrebov, Durand, Proudhon, Cailletaud. *C.R. Mécan.* (2011)

In theory

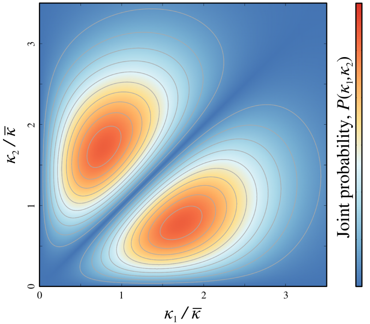
- Isotropy of surface does not imply isotropy of asperities
- Unbounded surface spectrum ⇒ divergence of mean curvature

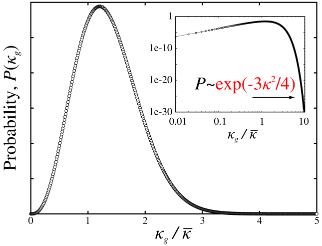
$$\bar{\kappa} = \sqrt{m_4} \sim k_s^{2-H} \xrightarrow{k_s \to \infty} \infty$$

[1] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[2] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[3] Greenwood, Wear 261: 191 (2006)

In reality

- continuum mechanics and fractal description fail at atomic scale
- brittle crystals → sharp corners (e.g. rocks, ceramics)

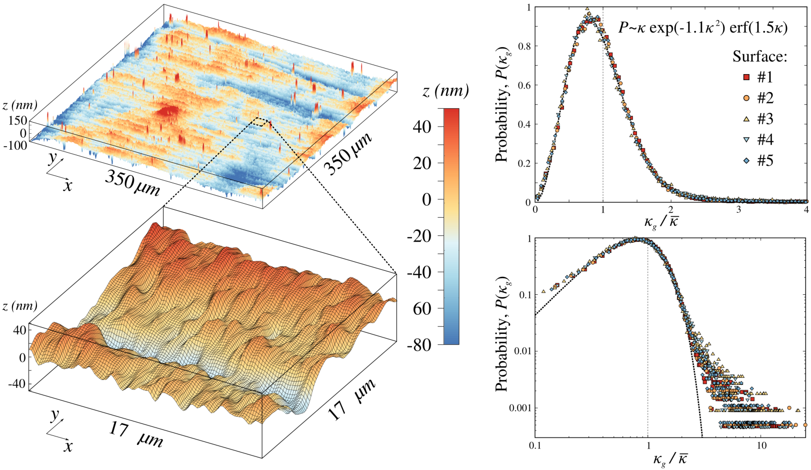




$P \sim \kappa^2 \exp(-3\kappa^2/4)\ \mathrm{erf}(3\kappa/2)$

• Example of real curvature distribution

• Roughness of CuZn electroplated with Ni (1 μm) and Au (1 μm)

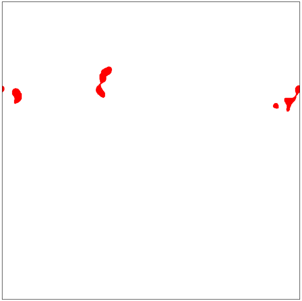


#Mechanics

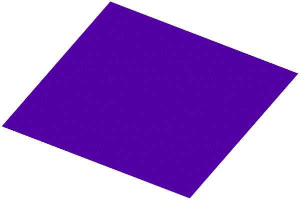
• Direct BEM / FEM analysis

- 3D simulations
- More or less accurate roughness representation
- Fast BEM[1,2,3]
  - *elastic*
  - *homogeneous*
- Slow FEM[4,5]
  - *arbitrary material model*
  - *geometrical-nonlinearity*
  - *heterogenity*

[1] Stanley & Kato. *J Tribol* (1997)
[2] Plonsky & Keer. *Wear* (1999)
[3] Liu, Wang, Liu. *Wear* (2000)
[4] Pei, Hyun, Molinari, Robbins. *J Mech Phys Solids* (2005)
[5] Yastrebov, Durand, Proudhon, Cailletaud. *CR Mecan* (2011)



True contact area



Contact pressure (zoom on 1/16 of the surface)
[6] Yastrebov, Anciaux, Molinari. *Int J Solids Struct* (2015)

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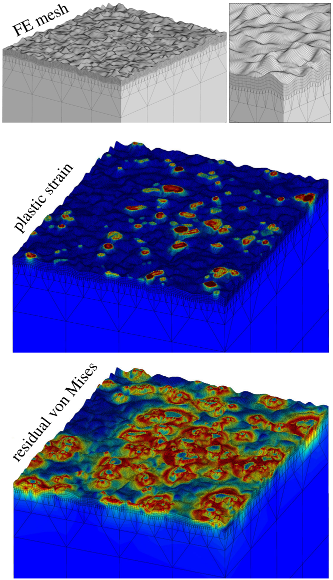

FE simulation of rough contact[5]

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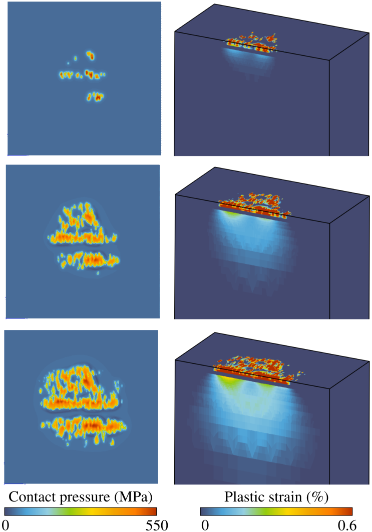


FE simulation electro-mechanical contact in system CuBe vs Au+Ni+CuZn
[7] Yastrebov et al, IEEE proceedings of Holm conference (2015).

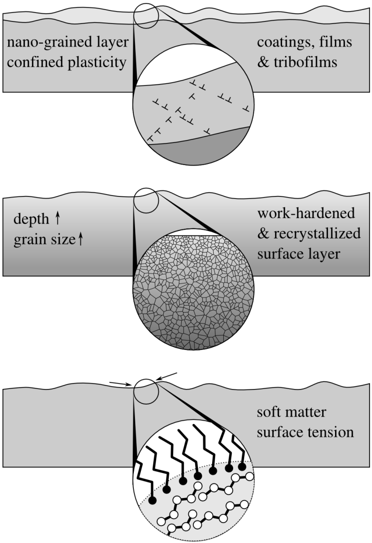
• Near-surface vs bulk deformation

Material aspects

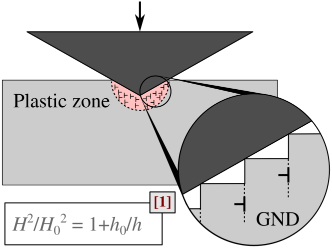
- Cold worked surface + recrystallized:
  smaller grains near the surface, Hall-Petch effect
- Thin coating films:
  nanograined, confined plasticity, Hall-Petch effect
- Oxides:
  brittle hard films

Geometrical aspects

- Roughness of all nature
- Indentation by asperities:
  confined plastic zone, high plastic strain gradients



• Indentation and hardness



[1] Nix, Gao. *J Mech Phys Solids* (1998).

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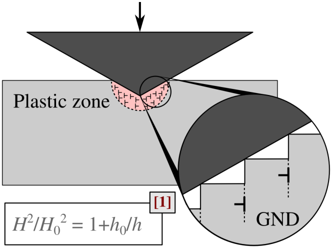

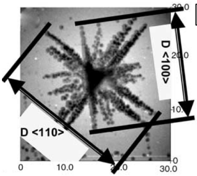

[2]

[1] Nix, Gao. *J Mech Phys Solids* (1998).
[2] Feng, Nix. *Scripta Mater* (2004).

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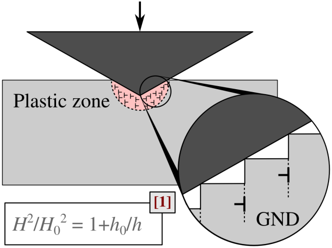


$$\boxed{H^2/H_0^2 = 1 + h_0/h}$$ [1]

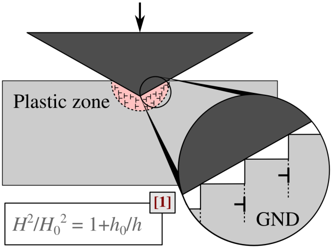
$$\frac{\text{Plastic zone } r}{\text{Contact radius } a} \sim 1 + b\,\exp(-h/h_1)$$

$$\boxed{H^2/H_0^2 = 1 + [1 + b\,\exp(-h/h_1)]^{-3} h_0/h}$$ [2]

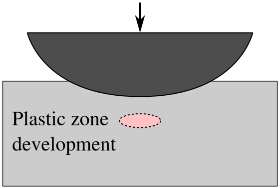
$$\boxed{(H - H_P)^2/(H_0 - H_P)^2 = 1 + [1 + b\,\exp(-h/h_1)]^{-3} h_0/h}$$ [2,3]

[1] Nix, Gao. *J Mech Phys Solids* (1998).
[2] Feng, Nix. *Scripta Mater* (2004).
[3] Qui, Huang, Nix, Hwang, Gao. *Acta Mater* (2001).

• Indentation and hardness



$$H^2/H_0^2 = 1 + h_0/h$$ [1]



$$\frac{\text{Plastic zone } r}{\text{Contact radius } a} \sim 1 + b\exp(-h/h_1)$$

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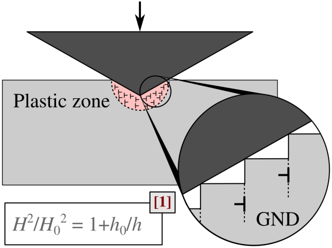
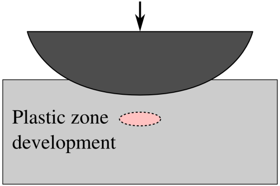


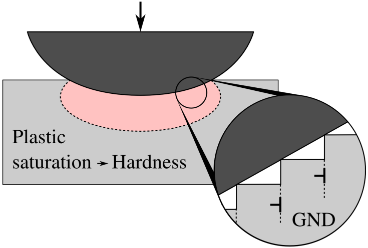
[1] $H^2/H_0^2 = 1 + h_0/h$

$$\frac{\text{Plastic zone } r}{\text{Contact radius } a} \sim 1 + b \exp(-h/h_1)$$

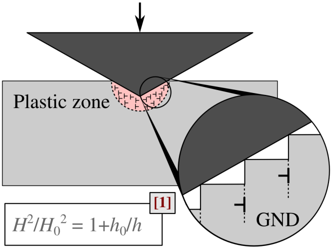
[2] $H^2/H_0^2 = 1 + [1 + b \exp(-h/h_1)]^{-3} h_0/h$

[2,3] $(H-H_P)^2/(H_0-H_P)^2 = 1 + [1 + b \exp(-h/h_1)]^{-3} h_0/h$





[1] Nix, Gao. *J Mech Phys Solids* (1998).
[2] Feng, Nix. *Scripta Mater* (2004).
[3] Qui, Huang, Nix, Hwang, Gao. *Acta Mater* (2001).

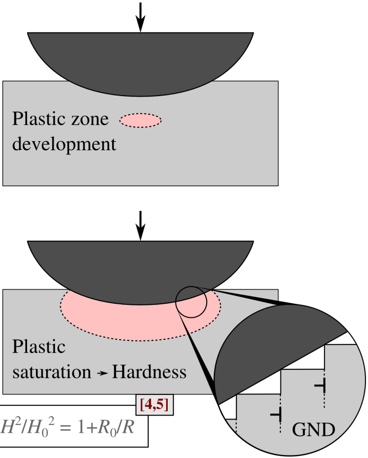


[1] $H^2/H_0^2 = 1+h_0/h$

$$\frac{\text{Plastic zone } r}{\text{Contact radius } a} \sim 1 + b\exp(-h/h_1)$$

[2] $H^2/H_0^2 = 1+[1+b\exp(-h/h_1)]^{-3}h_0/h$

[2,3] $(H-H_P)^2/(H_0-H_P)^2 = 1+[1+b\exp(-h/h_1)]^{-3}h_0/h$



[4,5] $H^2/H_0^2 = 1+R_0/R$

[1] Nix, Gao. *J Mech Phys Solids* (1998).
[2] Feng, Nix. *Scripta Mater* (2004).
[3] Qui, Huang, Nix, Hwang, Gao. *Acta Mater* (2001).
[4] Swadener, George, Pharr. *J Mech Phys Solids* (2002).
[5] Gao, Larson, Lee, Nicola, Tischler, Pharr. *J Appl Mech* (2015).

• Onset of yielding

Hertz contact: body of revolution

- Onset of plasticity for pressure

$$p_Y = 1.6\sigma_Y$$

- Associated force

$$F_Y = \frac{1.6^3\pi^3 R^2}{6}\left(\frac{\sigma_Y}{E^*}\right)^2 \sigma_Y$$

- Associated contact radius

$$a_Y = \frac{1.6\pi R}{2}\frac{\sigma_Y}{E^*}$$

- Plastic flow starts at depth

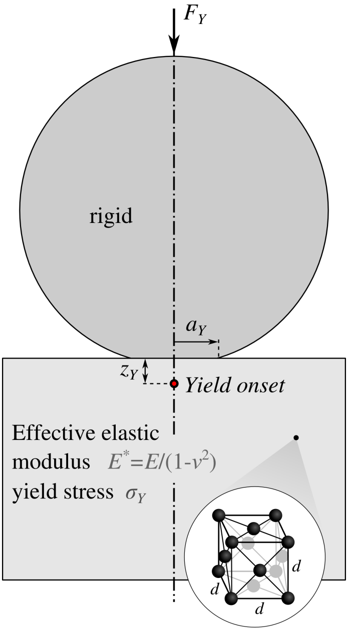
$$z_Y \approx 1.21 R \frac{\sigma_Y}{E^*}$$

- Example: golden asperity $R = 10\ \mu\text{m}$

$E^* \approx 96$ GPa, $\sigma_y \approx 140$ MPa, $d \approx 4.1$Å

$$\boxed{F_Y \approx 3.8\ \mu\text{N}, \quad z_Y \approx 18\ \text{nm}, \quad a_Y \approx 36\ \text{nm}}$$

$$z_Y \approx 45d, \quad a_Y \approx 115d$$



• Cosserat continuum

- Field variables (displacement & rotation): $\boldsymbol{u}, \boldsymbol{\omega}$
- Small deformation tensor: $\boldsymbol{\varepsilon} = \nabla \boldsymbol{u} + {}^3\boldsymbol{\epsilon} \cdot \boldsymbol{\omega}$
- Torsion-curvature tensor: $\boldsymbol{\kappa} = \nabla \boldsymbol{\omega}$
- Elasticity: $\boldsymbol{\sigma} = \lambda \mathrm{tr}\,(\boldsymbol{\varepsilon}_e)\,\boldsymbol{I} + \mu(\boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_e^\intercal) + \mu_c(\boldsymbol{\varepsilon}_e - \boldsymbol{\varepsilon}_e^\intercal), \quad \boldsymbol{m} = \alpha \mathrm{tr}\,(\boldsymbol{\kappa}_e)\,\boldsymbol{I} + 2\beta\boldsymbol{\kappa}_e$

  $$\boxed{l_e = \sqrt{\beta/\mu}}$$

  Note: $\boldsymbol{\varepsilon}^\intercal \neq \boldsymbol{\varepsilon}, \boldsymbol{\kappa}^\intercal \neq \boldsymbol{\kappa}, \boldsymbol{\sigma}^\intercal \neq \boldsymbol{\sigma}, \boldsymbol{m}^\intercal \neq \boldsymbol{m}$
- In non-inertial problems without volume forces and couple-forces, balance of momentum and of moment of momentum:

  $$\nabla \cdot \boldsymbol{\sigma} = 0, \quad \nabla \cdot \boldsymbol{m} - {}^3\boldsymbol{\epsilon} : \boldsymbol{\sigma} = 0$$
- Plasticity: equivalent stress[1,2] $\Upsilon = \sqrt{\frac{3}{2}\left(a_1 \boldsymbol{s} : \boldsymbol{s} + a_2 \boldsymbol{s} : \boldsymbol{s}^\intercal + \boxed{\frac{1}{l_p^2}}\,\boldsymbol{m} : \boldsymbol{m}\right)}$
- Internal lengths: elastic l_e, plastic l_p

[1] R. de Borst, L.J. Sluys, *Comp Meth Appl Mech Engin* (1991)
[2] S. Forest, R. Sievert, *Acta Mech* (2003)

where permutation tensor $^3\epsilon \sim \epsilon_{ijk} = \begin{cases} 1, & \text{if } \{ijk\} = \{123\} \text{ or } \{231\} \text{ or } \{312\} \\ -1, & \text{if } \{ijk\} = \{321\} \text{ or } \{213\} \text{ or } \{132\} \\ 0, & \text{otherwise} \end{cases}$

• Single asperity analysis

Assumptions

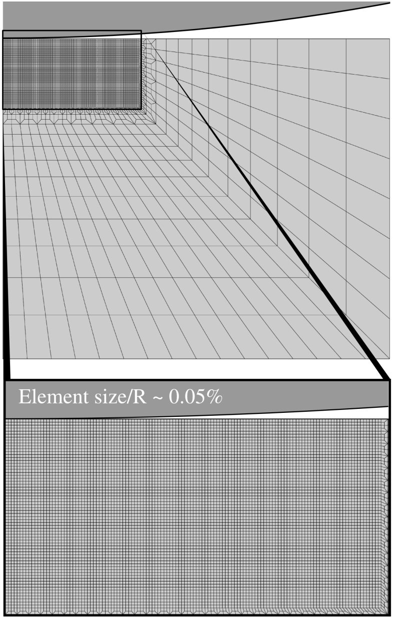
- Rigid spherical asperity
- Axisymmetric FE problem
- Generalized Cosserat continuum

Parameters

- Au: $E = 96$ GPa, $\nu = 0.42$, $\sigma_y = 140$ MPa
- $\mu_c = 10\mu, l_e = 100$ nm$, a_1 = 1$
- Indenter radius $R \in [0.002, 2000]$ μm

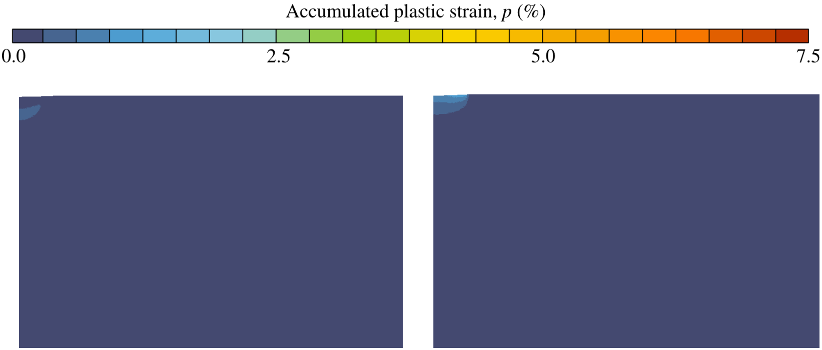
Objectives

- Study size effect
- Enhance asperity based models for rough contact



• Accumulated plasticity

- Different plastic distribution

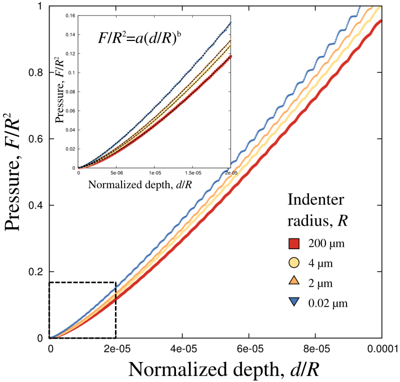


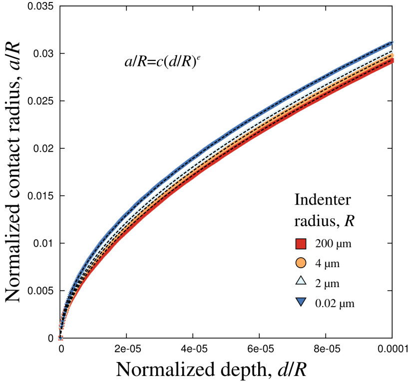
Displacement × 5 | Displacement × 5

Indenter radius $R = 20\mu m$
Max plastic strain $p_{\max} \approx 7.5\%$

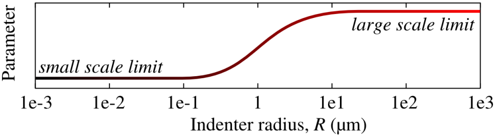
Indenter radius $R = 2\mu m$
Max plastic strain $p_{\max} \approx 11\%$

• Displacement–force–contact radius

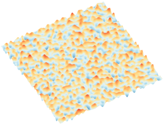


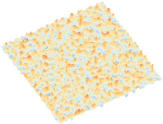


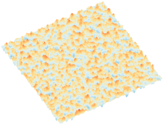
Parameters:
$a(R), b(R), c(R), e(R)$
`else:` using NURBS curve fit.



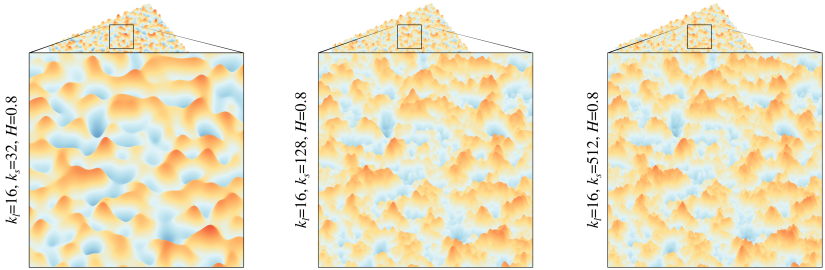
• Roughness → asperities



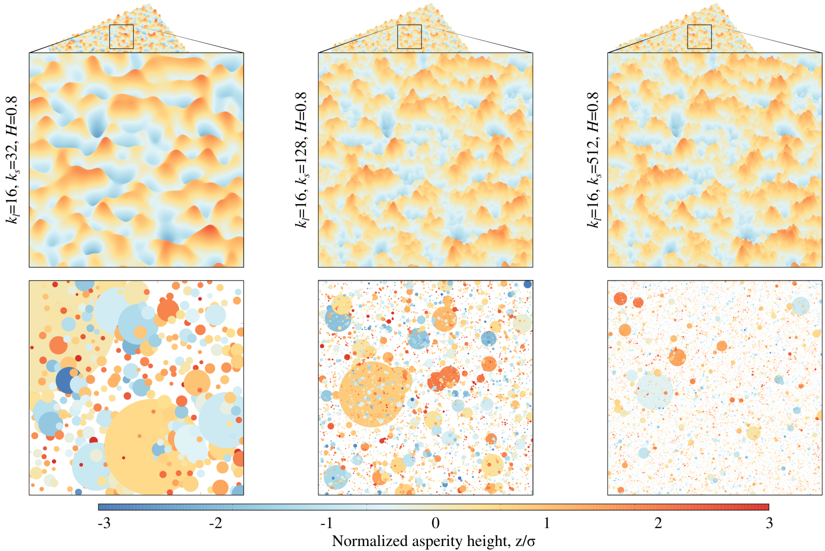




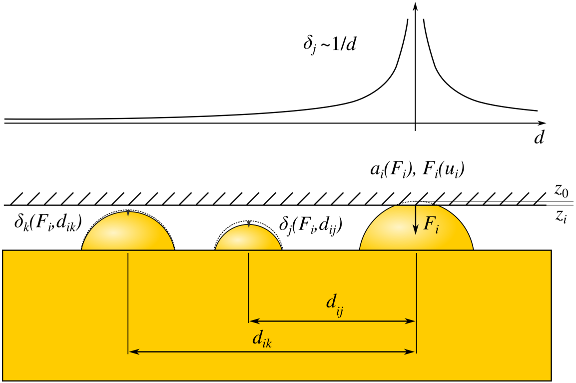
• Roughness → asperities



• Roughness → asperities



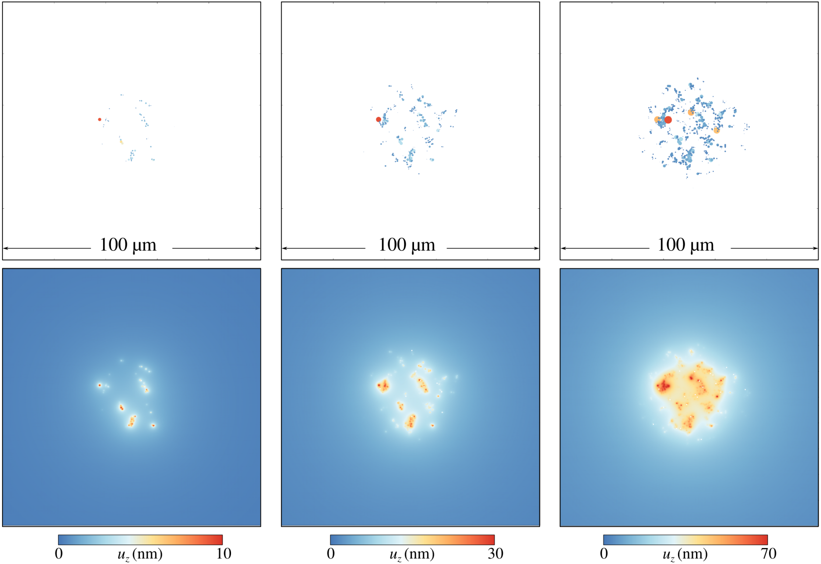
• Long-range elastic interaction



- Long-range interaction: $\delta_j = \frac{1-\nu^2}{\pi E} \sum\limits_{i=1}^{N} \frac{F_i}{d_{ij}}$
- Local indentation depth: $u_j = \max\{z_j - z_0 - \delta_j, 0\}$
- Force: $F_j = F(u_j)$

• Spherical indentation of a rough surface

- Indenter $R = 100\mu m$



• Conclusion & prospectives

First step towards incorporating scale dependent plasticity in contact behavior of rough surfaces.

Perspectives:

- Experimental validation[1]
- Frictional contact for Cosserat continuum[2,3]
- Second-gradient plasticity model[4,5]
- Clarify scale dependece of spherical indentation[5,6]

[1] Yastrebov, Mballa Mballa, Cailletaud, Noël, Houzé, Proudhon, Testé, IEEE Holm conference (2015).
[2] Zhang, Wang, Wriggers, Schrefler. *Comp Mech* (2005)
[3] Salehi, Salehi. *Int J Solids Struct* (2015)
[4] Cordero, Forest, Busso, Berbenni, Cherkaoui. *Comp Mater Sci* (2012)
[5] Gao, Larson, Lee, Nicola, Tischler, Pharr. *J Appl Mech* (2015)
[6] Swadener, George, Pharr. *J Mech Phys Solids* (2002)

First step towards incorporating scale dependent plasticity in contact behavior of rough surfaces.

Perspectives:

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- Frictional contact for Cosserat continuum[2,3]
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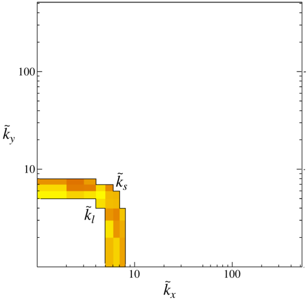
Thank you for your attention!

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



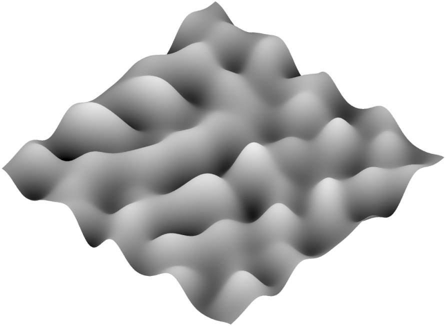


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 8$

- Effect of the high frequency cutoff k_s

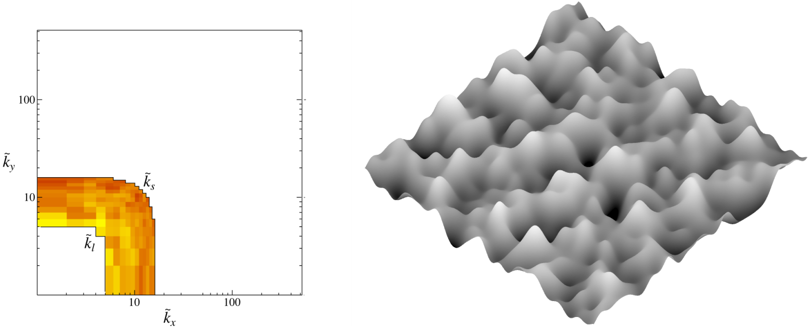
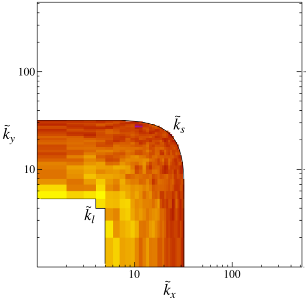


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 16$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



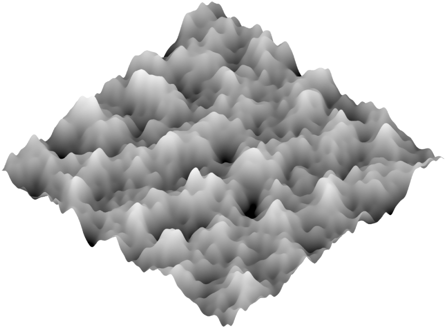
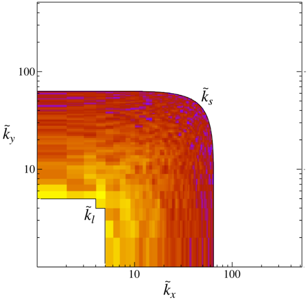


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 32$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



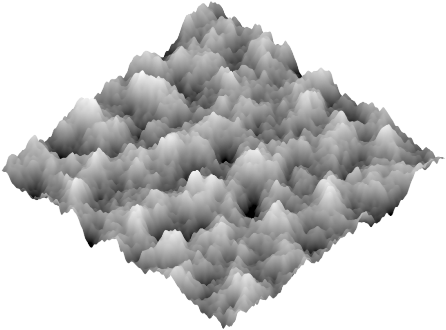


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 64$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

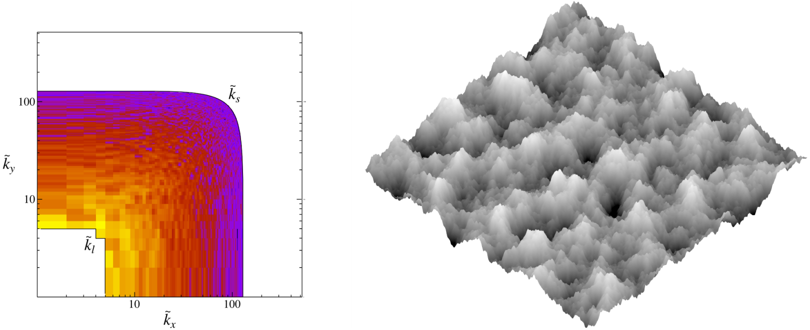


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 128$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for k_s/k_l = const

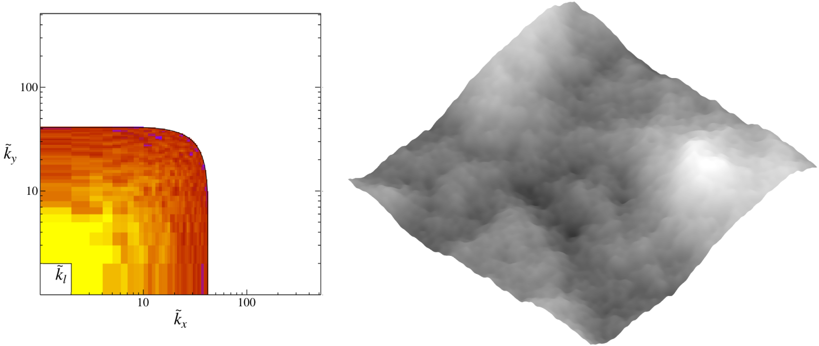


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 1, \quad k_s = 43$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \mathrm{const}$

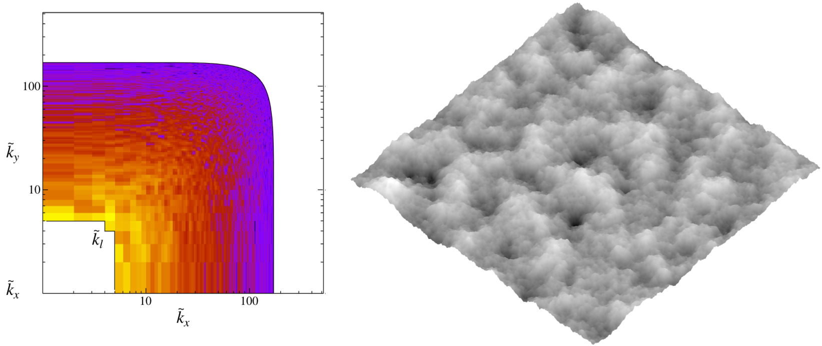


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 171$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for k_s/k_l = const

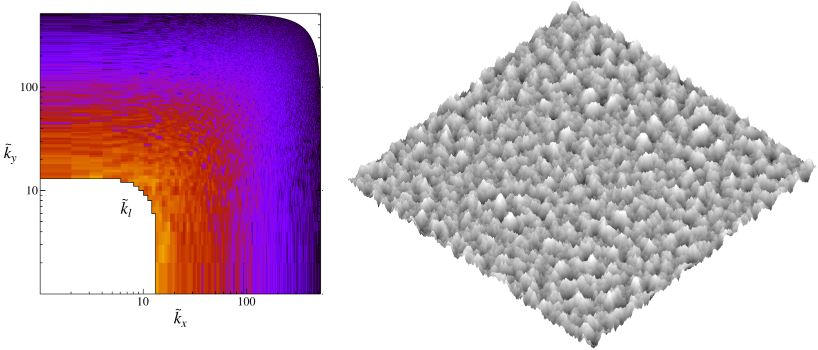


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for $k_l = 12, \quad k_s = 512$

Effect of parameters

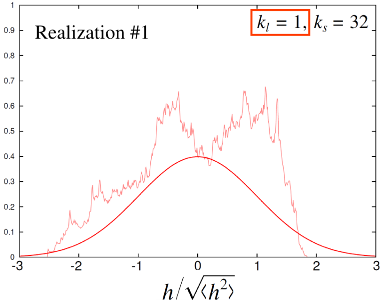
Effect of parameters:

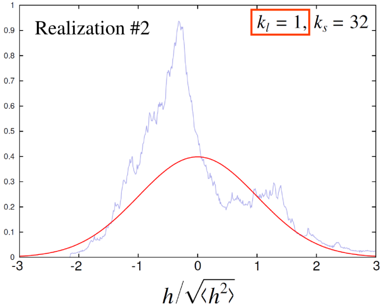
- k_l low frequency cutoff
  - representativity/normality[1,2,3]
- k_s high frequency cutoff
  - smoothness and density of asperities
- $\zeta = k_s/k_l$ ratio[3]
  - magnification

$$\boxed{\alpha \sim \zeta^{2H}}$$

Nayak's parameter α is the central characteristic of roughness in asperity based mechanical models.

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)
[3] Yastrebov, Anciaux, Molinari, Phys. Rev. E (2012)
[4] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. (2015)





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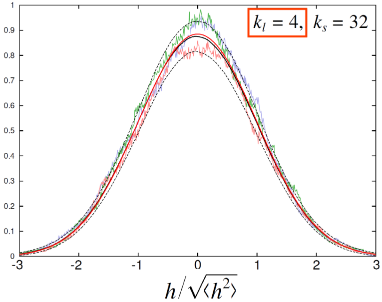
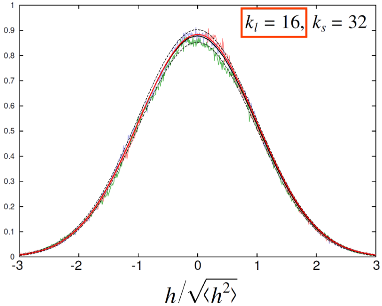
[4] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. (2015)





Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to mesh density $k_s/(N/L)$

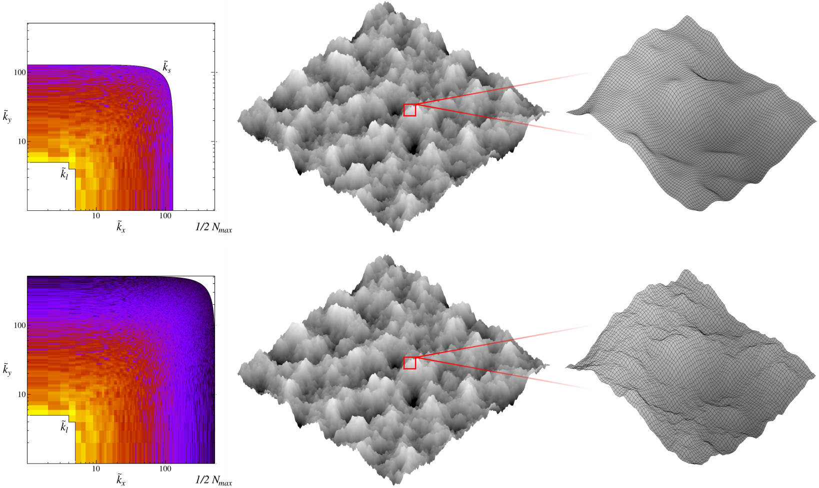


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
$k_l = 12, \quad k_s/N = 1/8$ **VS** $k_l = 12, \quad k_s/N = 1/2$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to mesh density $k_s/(N/L)$

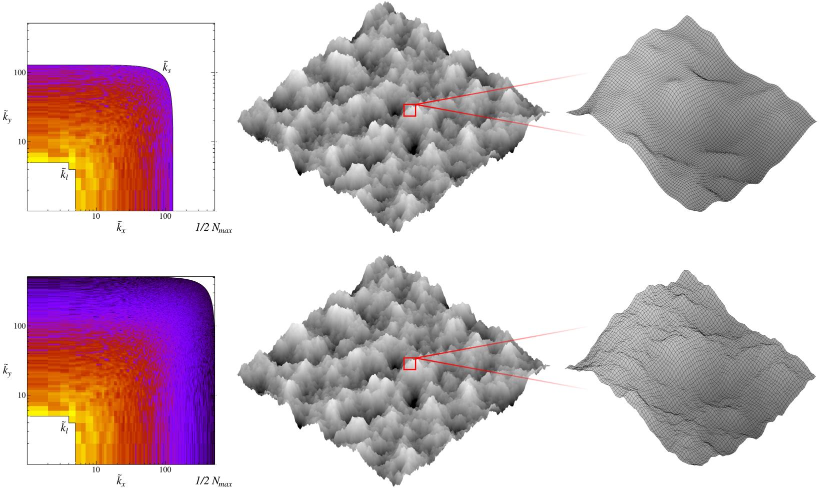


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
$k_l = 12$, $k_s/N = 1/8$ **(fine)** **VS** $k_l = 12$, $k_s/N = 1/2$ **(too coarse)**
for mechanical simulations

Effect of parameters: illustration

- Effect of the discretisation (single asperity)

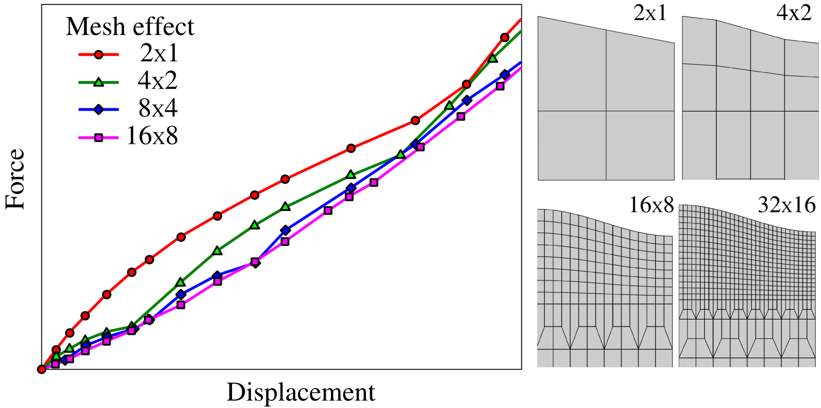


Fig. Effect of the mesh on mechanical response

Effect of parameters: illustration

- Data interpolation (Shanon, bi-cubic Bézier surfaces)

Experimental | Smoothed (enriched)

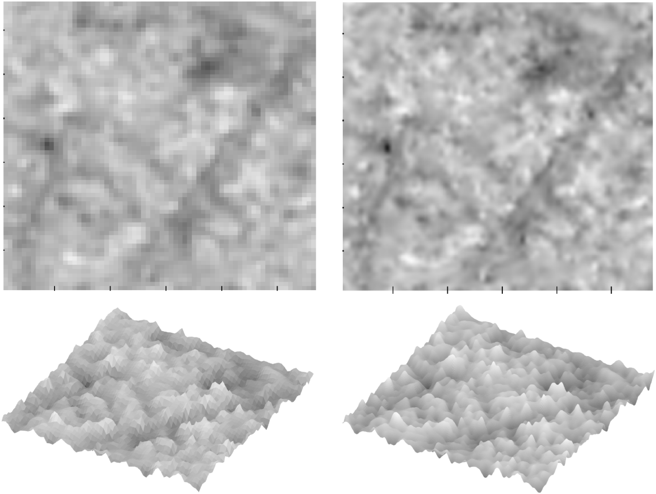


Fig. Bi-cubic Bézier interpolation of an experimental rough surface

[1]Hyun, Robbins. *Tribol. Int.* (2007)
[2] Yastrebov, Durand, Proudhon, Cailletaud. *C.R. Mécan.* (2011)